Joint Subcarrier Assignment and Power Allocation in Full-Duplex OFDMA Networks

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Abstract

Recent advances in communication technology have demonstrated the feasibility of full-duplex communication for a node to simultaneously transmit and receive on the same frequency band. Although the full-duplex technique has the potential to double the capacity of point-to-point wireless links, the resource allocation in multi-user environment becomes more complicated due to user-dependent channel conditions and per-user power constraints. In this paper, we formulate the joint problem of subcarrier assignment and power allocation in full-duplex OFDMA networks, and develop an iterative solution that achieves local Pareto optimality in typical scenarios. We evaluate our solution through extensive simulations, which shows that it empirically achieves near-optimal performance and outperforms other resource allocation schemes designed for half-duplex networks.

Index Terms

Full-duplex communications, OFDMA, wireless resource allocation.

I. Introduction

Half-duplex has been a most common assumption in wireless communications and restricts a node to either transmit or receive at a time [1]. Recent advances in signal processing have challenged this assumption and demonstrated the feasibility of wireless full-duplex communications, which enables a full-duplex node to transmit and receive simultaneously on the same frequency

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band by countervailing against the self-interference caused by its own transmission. Due to its potential to boost the wireless capacity, the full-duplex capability has received tremendous attention from both the academic and industrial world these days.

The main difficulty in full-duplex communications is suppressing self-interference to a sufficiently low level. In the literature, extensive researches have been conducted for self-interference cancellation techniques, which can be categorized into three groups: antenna cancellation, analog cancellation, and digital cancellation. In antenna cancellation, a pair of transmission antennas are placed such that the signal from one antenna is added destructively with the signal from the other [2], [3], [4]. Analog cancellation removes self-interference by subtracting the known transmission signal from the received signal through a noise cancellation circuit [2], [5]. Lastly, digital cancellation is used to clean out any remaining self-interference caused by non-ideal and non-linear components in RF chains [2], [5]. The state-of-the-art work has demonstrated that self-interference can be suppressed to the noise floor level by the combination of multiple cancellation techniques [6].

Orthogonal Frequency Division Multiple Access (OFDMA) has been a key technology for multiple access in most wide-band cellular networks [7], [8], [9], [10], which consist of a base station (BS) and multiple mobile nodes. By dividing the entire spectrum into multiple orthogonal subcarriers and distributing them over nodes, OFDMA can benefit from both multiuser and frequency diversities. To fully exploit such gains, it is essential to design a radio resource allocation solution that jointly optimizes subcarrier assignment and power allocation.

Resource management in (half-duplex) downlink or uplink OFDMA systems has been extensively studied in the literature to maximize the sum-rate by assigning subcarriers and allocating transmission power under a limited power budget. It has been known that an optimal solution to the downlink resource allocation problem is a combination of the channel-based subcarrier assignment and the well-known water-filling power allocation [11]. On the other hand, the uplink problem is more challenging due to per-node power constraints, i.e., each node of uplink transmission has its own power budget. Most previous results achieve suboptimal performance [12], [13], or solve a relaxed approximation problem [14].

The full-duplex capability enables the BS to transmit downlink traffic to nodes while receiving uplink traffic from them simultaneously. Since the uplink and downlink transmissions are no longer independent, previous solutions are unlikely to optimize the performance, thus necessitat-

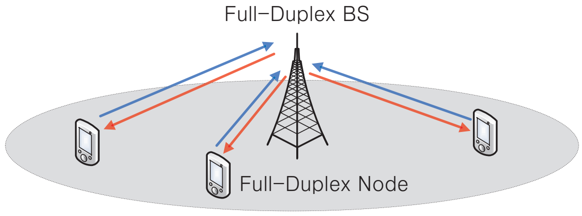


Fig. 1. A single-cell full-duplex OFDMA network which consists of one full-duplex base station (BS) and multiple full-duplex mobile nodes. Using the full-duplex capability, the BS exchanges uplink and downlink traffic with each node simultaneously.

ing new solutions that account for the characteristics of the simultaneous uplink and downlink transmissions. In this paper, we aim to maximize the full-duplex sum-rate by jointly optimizing subcarrier assignment and power allocation in the presence of full-duplex transmissions. The contributions of this paper can be summarized as follows:

- We propose a joint solution to the subcarrier assignment and power allocation problem by establishing a necessary condition for the sum-rate optimality.
- We show that our algorithm is provably efficient in achieving *local Pareto optimality* under certain conditions that are frequently met in practice.
- By decoupling uplink and downlink transmissions and ignoring inter-node interference, we characterize the optimal performance in polynomial time complexity.
- Through extensive simulations, we show that our algorithm empirically achieves near-optimal performance and outperforms other resource allocation schemes. Also, we verify the tightness of our analytical results under symmetric channel environments.

The rest of this paper is organized as follows. The full-duplex sum-rate maximization problem is formally formulated in Section II, and a necessary condition for optimality is derived in Section III. Our proposed subcarrier assignment and power allocation algorithm is described in Section IV, and its performance is analytically evaluated in Section V. We further characterize the full-duplex sum-rate and obtain a performance bound in Section VI, and empirically evaluate our solution in comparison with the bound and other resource allocation schemes in Section VII. Finally, we conclude our paper in Section IX.

II. System Model

We consider a single-cell full-duplex OFDMA network, as shown in Fig. 1. There are one full-duplex base station (BS) and N full-duplex mobile nodes, and let $\mathcal{N} = \{1, 2, \dots, N\}$ denote the set of nodes. The entire frequency band is partitioned into S subcarriers, and let $\mathcal{S} = \{1, 2, \ldots, S\}$ denote the set of subcarriers. Each subcarrier is perfectly orthogonal to each other without inter-subcarrier interference. We assume that self-interference can be successfully suppressed below the noise power level by various cancellation techniques [17]. Using the full-duplex capability, the BS exchanges downlink traffic and uplink traffic with each node simultaneously.

We assume that a subcarrier is exclusively assigned to a node for both uplink and downlink transmissions together. If a subcarrier were assigned to two different nodes (one node for uplink and the other for downlink), there would be substantial inter-node interference due to the uplink transmission [17], and the downlink transmission may not be acceptable. The optimal power allocation that takes into account the inter-node interference requires the channel information of the inter-node link, which is hard to obtain in practice due to a large feedback overhead. In this paper, we focus only on node-exclusive subcarrier allocation for both uplink and downlink transmissions.

We denote a subcarrier assignment pattern by a binary vector $X := \{x_{n,s}\}_{n \in \mathcal{N}, s \in \mathcal{S}}$, where element $x_{n,s}$'s are defined as

$$x_{n,s} = \begin{cases} 1, & \text{if subcarrier } s \text{ is assigned to node } n, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Let $u_{n,s}$ and $d_{n,s}$ denote the channel gains for subcarrier s (normalized with respect to the noise power) between the BS and node n for uplink and downlink, respectively, and let $G := \{u_{n,s}, d_{n,s}\}_{n \in \mathcal{N}, s \in \mathcal{S}}$ denote the channel gain vector. Also, $p_{n,s}$ and $q_{n,s}$ denote the uplink and downlink powers allocated to subcarrier s for node n, respectively. Let $P := \{p_{n,s}\}_{n \in \mathcal{N}, s \in \mathcal{S}}$ denote the uplink power allocation vector, and $Q := \{q_{n,s}\}_{n \in \mathcal{N}, s \in \mathcal{S}}$ denote the downlink power allocation vector. The total transmission powers at the BS and node n are limited to P_{BS} and P_n, respectively. From now on, we omit the subscript $n \in \mathcal{N}, s \in \mathcal{S}$ for brevity unless confusion arises.

The subcarrier sum-rate $R_{n,s}$ of node n in subcarrier s can be written as

$$\begin{aligned} & R_{n,s}(X, P, Q) \\ & = x_{n,s} \{\log(1 + p_{n,s} u_{n,s}) + \log(1 + q_{n,s} d_{n,s})\}. \end{aligned} \tag{2}$$

Let $R_n(X, P, Q)$ denote the sum-rate of node n over all subcarriers, i.e., $R_n(X, P, Q) =$

$\sum_{s=1}^{S} R_{n,s}$ and let $R(X,P,Q)$ denote the total sum-rate, i.e., $R(X,P,Q)=\sum_{n=1}^{N} R_n$.

In this paper, our goal is to maximize the total sum-rate $R(X,P,Q)$ by jointly optimizing the subcarrier assignment X and the power allocation (P,Q) under the power constraints of the BS and each node. Then we formally formulate the full-duplex sum-rate maximization problem P as follows:

$$(\mathsf{P})\ \text{maximize}\ R(X,P,Q) \tag{3}$$

$$\text{subject to}\ \sum_{n=1}^{N} x_{n,s} \leq 1, \forall s \in \mathcal{S} \tag{4}$$

$$\sum_{s=1}^{S} p_{n,s} \leq P_n, \forall n \in \mathcal{N} \tag{5}$$

$$\sum_{n=1}^{N}\sum_{s=1}^{S} q_{n,s} \leq P_{BS}, \tag{6}$$

$$p_{n,s} \geq 0, \forall n \in \mathcal{N}, \forall s \in \mathcal{S} \tag{7}$$

$$q_{n,s} \geq 0, \forall n \in \mathcal{N}, \forall s \in \mathcal{S} \tag{8}$$

$$x_{n,s} \in \{0,1\}, \forall n \in \mathcal{N}, \forall s \in \mathcal{S}. \tag{9}$$

Note that each subcarrier is exclusively assigned to a single node according to (4) and (9).

III. Necessary Condition for Optimality

Due to the exclusive nature of subcarrier assignment, the original problem P is an integer optimization problem, which generally requires exponential complexity to be solved. Therefore, we relax the constraints and allow multiple nodes to share a subcarrier together. The binary constraints (9) are replaced with

$$x_{n,s} \geq 0, \forall n \in \mathcal{N}, \forall s \in \mathcal{S}. \tag{10}$$

From (4) and (10), we have $x_{n,s} \in [0,1]$.

The relaxed problem P' obtained by replacing (9) with (10) is still not a convex problem because $R(X,P,Q)$ is not (jointly) concave in X and (P,Q). However, the optimal solution still satisfies the Karush-Kuhn-Tucker (KKT) condition [18], so we can obtain the following proposition.

Proposition 1. Let $X^* = \left\{x^*_{n,s}\right\}$, $P^* = \left\{p^*_{n,s}\right\}$, and $Q^* = \left\{q^*_{n,s}\right\}$ denote the optimal solution to problem P'. Then X^* and (P^*, Q^*) satisfy the following conditions:

1) For $x^*_{n,s} > 0$,

$$n = \arg\max_{m \in \mathcal{N}} \left\{\log\left(1 + p_{m,s} u_{m,s}\right) + \log\left(1 + q_{m,s} d_{m,s}\right)\right\}. \tag{11}$$

2) For $p^*_{n,s} > 0$,

$$s = \arg\max_{l \in \mathcal{S}} \left(\frac{x_{n,l} u_{n,l}}{1 + p_{n,l} u_{n,l}}\right). \tag{12}$$

3) For $q^*_{n,s} > 0$,

$$(n, s) = \arg\max_{(m \in \mathcal{N}, l \in \mathcal{S})} \left(\frac{x_{m,l} d_{m,l}}{1 + q_{m,l} d_{m,l}}\right). \tag{13}$$

Proof: We can define the Lagrangian function L as

$$\begin{aligned} & L(X, P, \lambda, \mu, \nu) \\ & := \sum_{n=1}^{N} \sum_{s=1}^{S} x_{n,s} \left\{\log\left(1 + p_{n,s} u_{n,s}\right) + \log\left(1 + q_{n,s} d_{n,s}\right)\right\} \\ & + \sum_{s=1}^{S} \lambda_s \left(1 - \sum_{n=1}^{N} x_{n,s}\right) + \sum_{n=1}^{N} \mu_n \left(P_n - \sum_{s=1}^{S} p_{n,s}\right) \\ & + \nu \left(P_{BS} - \sum_{n=1}^{N} \sum_{s=1}^{S} q_{n,s}\right), \end{aligned} \tag{14}$$

where $\lambda = \{\lambda_s\}$, $\mu = \{\mu_n\}$, and ν are the dual variables (or the Lagrangian multipliers). Then X^* and (P^*, Q^*) should satisfy the following KKT conditions:

$$\begin{aligned} \left.\frac{\partial L}{\partial x_{n,s}}\right|_{X^*, P^*, Q^*} = & \log\left(1 + p^*_{n,s} u_{n,s}\right) + \log\left(1 + q^*_{n,s} d_{n,s}\right) \\ & - \lambda_s \begin{cases} = 0, \text{if } x^*_{n,s} > 0 \\ \leq 0, \text{if } x^*_{n,s} = 0, \end{cases} \end{aligned} \tag{15}$$

$$\left.\frac{\partial L}{\partial p_{n,s}}\right|_{X^*, P^*, Q^*} = \frac{x^*_{n,s} u_{n,s}}{1 + p^*_{n,s} u_{n,s}} - \mu_n \begin{cases} = 0, \text{if } p^*_{n,s} > 0 \\ \leq 0, \text{if } p^*_{n,s} = 0, \end{cases} \tag{16}$$

$$\left.\frac{\partial L}{\partial q_{n,s}}\right|_{X^*, P^*, Q^*} = \frac{x^*_{n,s} d_{n,s}}{1 + q^*_{n,s} d_{n,s}} - \nu \begin{cases} = 0, \text{if } q^*_{n,s} > 0 \\ \leq 0, \text{if } q^*_{n,s} = 0. \end{cases} \tag{17}$$

From (15), if subcarrier s is assigned to node n (i.e., $x^*_{n,s} > 0$), node n has the largest subcarrier sum-rate $R_{n,s} = \lambda_s$, which implies (11). Similarly, we can obtain eq. (12) from (16) and eq. (13) from (17), respectively. ■

Although Proposition 1 gives a necessary condition for optimality, we cannot directly obtain an optimal solution because the conditions for X^* and (P^*, Q^*) are interdependent. Instead, we can have an intuition from (11) that each subcarrier s should be allocated to node n with the largest subcarrier sum-rate. Motivated by this intuition, we propose a resource allocation algorithm next.

IV. Proposed Resource Allocation Algorithm

In this section, we develop a solution that blends greedy subcarrier assignment and water-falling power allocation under per-node power constraints. We start with describing power allocation, and then consider subcarrier assignment.

A. Power Allocation

Given an optimal subcarrier assignment X^*, problem P is reduced to a power allocation problem that can be easily solved by the well-known *water-filling* power allocation at the BS and each node. Specifically, each node n allocates its uplink power $p_{n,s}$ to the assigned subcarrier s as

$$p_{n,s} = \begin{cases} [\alpha_n - 1/u_{n,s}]^+, \text{if } x^*_{n,s} = 1 \\ 0, \qquad\qquad\qquad\quad \text{if } x^*_{n,s} = 0, \end{cases} \tag{18}$$

where $[\cdot]^+ := \max\{\cdot, 0\}$ and α_n is a constant (called water level) satisfying $\sum_{s=1}^{S} p_{n,s} = P_n$. Similarly, the BS can optimally allocate the downlink power $q_{n,s}$ to subcarrier s for node n with $x^*_{n,s} = 1$ as

$$q_{n,s} = \begin{cases} [\alpha - 1/d_{n,s}]^+, \text{if } x^*_{n,s} = 1 \\ 0, \qquad\qquad\qquad \text{if } x^*_{n,s} = 0, \end{cases} \tag{19}$$

where α is a constant satisfying $\sum_{n=1}^{N} \sum_{s=1}^{S} q_{n,s} = P_{BS}$.

B. Subcarrier Assignment

Next, we design the subcarrier assignment algorithm that assigns subcarriers in an iterative manner, one-by-one, to a node with the largest subcarrier sum-rate. We take into account the dependency on the transmission power by re-allocating the power in each iteration. The detailed algorithm is provided in Algorithm 1.

Let $\mathcal{S}_n^{(k)}$ denote the set of subcarriers assigned to node n up to iteration k. If subcarrier s is assigned to a node, we denote the node by $n(s)$. Let $\mathcal{A}^{(k)}$ denote the set of assigned subcarriers up to iteration k, i.e., $\mathcal{A}^{(k)} = \cup_n \mathcal{S}_n^{(k)}$, and let $\mathcal{U}^{(k)} = \mathcal{S} \backslash \mathcal{A}^{(k)}$. We also use the superscript (k) to denote new power allocations, sum-rates, and downlink channel gains, calculated in iteration k. We use the superscript (0) to denote the initial value, e.g., $\mathcal{A}^{(0)} = \emptyset$.

In iteration k, we compute the potential subcarrier sum-rate given the subcarrier assignment of up to iteration ($k-1$), and select a pair of (node, subcarrier) with the largest subcarrier sum-rate. To further elaborate, we first compute the subcarrier sum-rates $R_{n,s}^{(k)}$ for each node n as follows:

1) For each node n, re-allocate its uplink power $p_{n,s}^{(k)}$ using the water-filling algorithm (18) to the subcarriers that are assigned to node n or unassigned, i.e., $\mathcal{S}_n^{(k-1)} \cup \mathcal{A}^{(k-1)}$.
2) For each node n, reset its downlink channel gain $d_{n,s}^{(k)} = d_{n(s),s}$ for assigned subcarrier $s \in \mathcal{A}^{(k-1)}$, and $d_{n,s}^{(k)} = d_{n,s}$ for unassigned subcarrier $s \in \mathcal{U}^{(k-1)}$. Then, allocate all the BS power $q_{n,s}^{(k)}$ to node n with channel gain $d_{n,s}^{(k)}$, using the water-filling algorithm (19).
3) For each node n, compute the (potential) subcarrier sum-rate as

$$R_{n,s}^{(k)} = \log\left(1 + p_{n,s}^{(k)} u_{n,s}\right) + \log\left(1 + q_{n,s}^{(k)} d_{n,s}^{(k)}\right). \tag{20}$$

Given the per-node per-subcarrier sum-rate (20), among the unassigned subcarriers, we find a pair of (node, subcarrier) for subcarrier assignment according to the following procedures:

1) $(n^*, s^*) = \underset{n \in \mathcal{N}, s \in \mathcal{U}^{(k-1)}}{\arg\max} R_{n,s}^{(k)}$.
2) $\hat{x}_{n^*,s^*} \leftarrow 1$.
3) Update $\mathcal{U}^{(k)} \leftarrow \mathcal{U}^{(k-1)} \backslash \{s^*\}$, $\mathcal{A}^{(k)} \leftarrow \mathcal{A}^{(k-1)} \cup \{s^*\}$, and $\mathcal{S}_{n^*}^{(k)} \leftarrow \mathcal{S}_{n^*}^{(k-1)} \cup \{s^*\}$.

We repeat the above procedures S times and obtain the subcarrier assignment vector $\hat{X}$. Given $\hat{X}$, we finally allocate some powers for each node and the BS by (18) and (19), and obtain the power allocation vector $(\hat{P}, \hat{Q})$.

In each iteration, we perform the water-filling for (uplink and downlink) power allocation for each node, which has the complexity of $O(S)$ [12]. Considering N nodes and S iterations, our solution has the complexity of $O(S^2 N)$. Also, our solution runs at the BS, and given the channel gains G, it returns $\hat{X}$ and $(\hat{P}, \hat{Q})$ as the final outcomes.

Algorithm 1: Full-Duplex Resource Allocation Algorithm

Data: S subcarriers, denoted as $\mathcal{S} = \{1, 2, \dots, S\}$.
N nodes, denoted as $\mathcal{N} = \{1, 2, \dots, N\}$.
channel gains $G = \{u_{n,s}, d_{n,s}\}$.
maximum power constraints $\{P_n\}$ and P_{BS}.

Initialization
  Set $\mathcal{A}^{(0)} \leftarrow \emptyset$;
  Set $\mathcal{U}^{(0)} \leftarrow \mathcal{S}$;
  Set $\mathcal{S}_n^{(0)} \leftarrow \emptyset$ for $\forall n \in \mathcal{N}$;

for $k = 1 \to S$ **do**
  for $n = 1 \to N$ **do**
    [Uplink power in iteration k]
      Allocate uplink power $p_{n,s}^{(k)}$ by (18) to the subcarriers that are assigned to node n or unassigned;
    [Downlink power in iteration k]
      Reset $d_{n,s}^{(k)} = d_{n(s),s}$ for assigned subcarrier $s \in \mathcal{A}^{(k-1)}$, and $d_{n,s}^{(k)} = d_{n,s}$ for unassigned subcarrier $s \in \mathcal{U}^{(k-1)}$;
      Allocate all the BS power $q_{n,s}^{(k)}$ by (19) to node n with channel gain $d_{n,s}^{(k)}$;
    [Subcarrier sum-rate in iteration k]
      Compute the potential subcarrrier sum-rate
      $R_{n,s}^{(k)} = \log\left(1 + p_{n,s}^{(k)} u_{n,s}\right) + \log\left(1 + q_{n,s}^{(k)} d_{n,s}^{(k)}\right)$;
  [Subcarrier assignment in iteration k]
    Find (n^*, s^*) such that $(n^*, s^*) = \underset{\left(n \in \mathcal{N}, s \in \mathcal{U}^{(k-1)}\right)}{\arg\max} \left(R_{n,s}^{(k)}\right)$;
    Set $\hat{x}_{n^*,s^*} \leftarrow 1$;
    Set $n_{s^*} \leftarrow n^*$;
    Update $\mathcal{S}_{n^*}^{(k)} \leftarrow \mathcal{S}_{n^*}^{(k-1)} \cup \{s^*\}$;
    Update $\mathcal{A}^{(k)} \leftarrow \mathcal{A}^{(k-1)} \cup \{s^*\}$;
    Update $\mathcal{U}^{(k)} \leftarrow \mathcal{U}^{(k-1)} \backslash \{s^*\}$;

[Power allocation]
  Given $\hat{x}_{n,s}$, allocate uplink power $\hat{p}_{n,s}$ by (18) and downlink power $\hat{q}_{n,s}$ by (19);

Result: Subcarrier assignment $\hat{X}$ and power allocation $(\hat{P}, \hat{Q})$.

V. Local Pareto Optimality

In this section, we show that our subcarrier assignment algorithm is provably efficient in achieving a certain optimal property. We start with several definitions related to local Pareto optimality [13].

Let $\mathcal{X}$ denote the set of all feasible subcarrier assignments satisfying the constraints (4) and (9). Given a feasible subcarrier assignment $X \in \mathcal{X}$, let $\mathcal{S}_n^X$ denote the set of subcarriers assigned to node n.

Definition 1. We define the distance $D(X, Y)$ between two feasible subcarrier assignments $X, Y \in \mathcal{X}$ as

$$D(X, Y) := \max_{n \in \mathcal{N}} \{d_n(X, Y)\}, \tag{21}$$

where

$$d_n(X, Y) := \max \left\{ |\mathcal{S}_n^X \backslash \mathcal{S}_n^Y|, |\mathcal{S}_n^Y \backslash \mathcal{S}_n^X| \right\}. \tag{22}$$

By definition, if subcarrier assignment is changed from X to Y, each node can win or lose at most $D(X, Y)$ subcarriers. It can be shown that $< \mathcal{X}, D >$ is a metric space, in which *ϵ-ball* is defined as follows: [13]

Definition 2. The *ϵ-ball* $B(X, \epsilon)$ of a feasible subcarrier assignment X is defined as

$$B(X, \epsilon) := \{Y \in \mathcal{X} | D(X, Y) \leq \epsilon\}. \tag{23}$$

Clearly, $B(X, \epsilon)$ is the set of subcarrier assignments whose distance to X is no greater than ϵ.

Given a feasible subcarrier assignment X, let (P^X, Q^X) denote the power allocation vector by the water-filling and R_n^X denote the sum-rate of node n, i.e., $R_n^X = R_n(X, P^X, Q^X)$.

We now introduce the following definitions of *Pareto domination* and *local Pareto optimality* [13].

Definition 3. A subcarrier assignment X *Pareto dominates* another subcarrier assignment Y if $R_n^X \geq R_n^Y$ for every node $n \in \mathcal{N}$, and $R_m^X > R_m^Y$ for at least one node $m \in \mathcal{N}$.

Definition 4. A subcarrier assignment X is *local Pareto optimal* in $B(X, \epsilon)$ if there is no subcarrier assignment $Y \in B(X, \epsilon)$ that Pareto dominates X.

Thus, if X is local Pareto optimal in $B(X, \epsilon)$, we cannot (strictly) increase the sum-rate of every node n by adding/removing at most ϵ subcarriers to/from $\mathcal{S}_n^X$.

To obtain some properties of our solution, we need the following three conditions.

Condition 1 (All-positive power allocation)**.** Either positive uplink power or positive downlink power is allocated to each subcarrier, i.e.,

$$\sum_{n=1}^{N}(p_{n,s}+q_{n,s})>0,\forall s\in\mathcal{S}. \tag{24}$$

Condition 2 (Elementwise-unique channel gain)**.** For each node n, each subcarrier has a distinctive channel gain both in uplink and downlink, i.e.,

$$u_{n,s}\neq u_{n,l},\forall s,l\in\mathcal{S}, \tag{25}$$

$$d_{n,s}\neq d_{n,l},\forall s,l\in\mathcal{S}. \tag{26}$$

Since $u_{n,s}$'s (and also $d_{n,s}$'s) are continuous real random variables, the probability that any two of them have an equal value is zero. Therefore, we can assume that a channel gain vector is elementwise-unique in practice.

Condition 3 (reciprocity-in-order)**.** For each node n, for any two subcarriers s and l, $u_{n,s}>u_{n,l}$ implies $d_{n,s}>d_{n,l}$ and vice versa, i.e., $u_{n,s}>u_{n,l}\leftrightarrow d_{n,s}>d_{n,l}$. This means that the order of subcarriers in terms of uplink channel gain is equal to their order in terms of downlink channel gain.

Note that two channels are said to be reciprocal (channel reciprocity) when the uplink channel gain and the downlink channel gain for each subcarrier are the same, i.e., $u_{n,s}=d_{n,s},\forall n,s$. The reciprocal channels are reciprocal-in-order (but the reverse does not hold in general).

Finally, we show the local Pareto optimality of our solution. From now on, we assume a channel gain vector $G=\{u_{n,s},d_{n,s}\}$ satisfying conditions 2 and 3. Let us choose a subcarrier assignment Y in $B(X,1)$. That is, Y can be obtained from X by reassigning subcarriers such that each node wins and/or loses at most 1 subcarrier. Assume that (P^X,Q^X) satisfies condition 1. The following lemma shows a necessary condition under which Y Pareto dominates X.

Lemma 1. For two subcarrier assignments X and $Y\in B(X,1)$ such that Y Pareto dominates X, when subcarrier assignment is changed from X to Y, every node either retains its subcarrier assignment unchanged or swaps one subcarrier for another one with a larger channel gain.

Proof: When subcarrier assignment is changed from X to $Y \in B(X,1)$, if there exists either a node that loses a subcarrier (Lemma 3 in Appendix 1) or swaps a subcarrier for another one with a smaller channel gain (Lemma 4 in Appendix 1), Y does not Pareto dominate X. The proof of Lemma 1 is straightforward using Lemma 3 and Lemma 4, which is provided in Appendix 1. ■

In addition, the following lemma shows a useful property of our algorithm.

Lemma 2. By our algorithm, when node n is assigned subcarrier s, it has the largest (uplink and downlink) channel gain in subcarrier s among all unassigned subcarriers.

Proof: We prove this by contradiction. Suppose that subcarrier s is assigned to node n in iteration k. This means that $R_{n,s}^{(k)} > R_{n,l}^{(k)}, \forall l \in \mathcal{U}^{(k-1)} \backslash \{s\}$. Now, assume that in the beginning of iteration k, there is subcarrier $l \in \mathcal{U}^{(k-1)}$ with $u_{n,l} > u_{n,s}$ and also $d_{n,l} > d_{n,s}$. Now, let $\alpha_n^{(k)}$ denote the uplink water-level of node n, and $\alpha^{(k)}$ denote the downlink water level. Then the subcarrier sum-rates $R_{n,s}^{(k)}$ and $R_{n,l}^{(k)}$ are given as

$$R_{n,s}^{(k)} = \left[\alpha_n^{(k)} - \frac{1}{u_{n,s}}\right]^+ + \left[\alpha^{(k)} - \frac{1}{d_{n,s}}\right]^+, \tag{27}$$

$$R_{n,l}^{(k)} = \left[\alpha_n^{(k)} - \frac{1}{u_{n,l}}\right]^+ + \left[\alpha^{(k)} - \frac{1}{d_{n,l}}\right]^+. \tag{28}$$

Since $u_{n,l} > u_{n,s}$ and $d_{n,l} > d_{n,s}$, we have $R_{n,l}^{(k)} \geq R_{n,s}^{(k)}$, which contradicts $R_{n,s}^{(k)} > R_{n,l}^{(k)}$. ■

Based on the above lemmas, we can prove that $\hat{X}$ is local Pareto optimal in its 1-ball if $(\hat{P}, \hat{Q})$ is an all-positive power allocation vector.

Theorem 1. Given a channel gain vector G satisfying conditions 2 and 3, if $(\hat{P}, \hat{Q})$ satisfies condition 1, $\hat{X}$ is local Pareto optimal in its 1-ball.

Proof: We prove this by contradiction. Assume that a subcarrier assignment $Y \in B(\hat{X}, 1)$ Pareto dominates $\hat{X}$. Then from Lemma 1, each node swaps a subcarrier for another one with a larger channel gain. Without loss of generality, assume that when subcarrier assignment is changed from $\hat{X}$ to Y, subcarrier s_n is reallocated from node n to node $n+1$ for $1 \leq n \leq M-1$, and subcarrier s_M from node M to node 1. In this case, we have $u_{n,s_{n-1}} > u_{n,s_n}$ and $d_{n,s_{n-1}} > d_{n,s_n}$ for $2 \leq n \leq M$, and $u_{1,s_M} > u_{1,s_1}$ and $d_{1,s_M} > d_{1,s_1}$.

Let k_n $(1 \le n \le M)$ denote the iteration index when subcarrier s_n is assigned to node n. By Lemma 2, node 1 has the largest channel gain in subcarrier s_1 among all unassigned subcarriers in iteration k_1. This implies that subcarrier s_M is assigned to node M before iteration k_1, i.e.,

$$k_1 > k_M. \tag{29}$$

Similarly, from node n's point of view $(2 \le n \le M)$, we have

$$k_2 > k_1, \tag{30}$$

$$k_3 > k_2, \tag{31}$$

$$\vdots$$

$$k_M > k_{M-1}. \tag{32}$$

Then we have a contradiction in the relation between $k_1, \cdots, k_M$. ■

Remark 1. When all the conditions do not hold, we can find a counterexample where our solution $\hat{X}$ is not local Pareto optimal in $B(\hat{X}, 1)$. We show later by simulations that condition 1 is frequently met in practice. Although the techniques we used above are similar to those in [13], there are important technical differences: when subcarrier assignment is changed from $\hat{X}$ to Y, the downlink rate of each node n must be carefully considered because it depends on the channel gains of other nodes as well as node n. For details, refer to Appendix 1.

VI. Performance Bound

In this section, we provide a performance bound by considering uplink and downlink transmissions separately.

For the original problem P, we assume that a subcarrier is used by only one node for a full-duplex link. We relax this constraint and assume that two different nodes can share a single subcarrier: one for uplink and the other for downlink. Now, we introduce two assignment variables $x_{n,s}$ and $y_{n,s}$ for downlink and uplink, respectively, defined as follows:

$$x_{n,s} = \begin{cases} 1, & \text{if subcarrier } s \text{ is assigned to node } n \\ & \text{in downlink transmission} \\ 0, & \text{otherwise.} \end{cases} \tag{33}$$

$$y_{n,s} = \begin{cases} 1, & \text{if subcarrier } s \text{ is assigned to node } n \\ & \text{in uplink transmission} \\ 0, & \text{otherwise.} \end{cases} \tag{34}$$

Also, $x_{n,s}$ and $y_{n,s}$ should satisfy $\sum_{n=1}^{N} x_{n,s} \leq 1, \forall s \in \mathcal{S}$, and $\sum_{n=1}^{N} y_{n,s} \leq 1, \forall s \in \mathcal{S}$, respectively. By replacing (9) with (33) and (34), we have another problem $\mathsf{P_R}$. Note that the optimal solution to P is still feasible in $\mathsf{P_R}$, i.e., $x^*_{n,s} = 1$ in P can be mapped to $x_{n,s} = y_{n,s} = 1$ in $\mathsf{P_R}$. Thus, the optimal solution to $\mathsf{P_R}$ achieves an upper bound of P.

When two different nodes use a subcarrier, the maximum achievable subcarrier sum-rate[1] is less than the sum of (point-to-point) uplink rate and downlink rate because the uplink transmission can cause severe inter-node interference at the downlink node. By assuming that there is no inter-node interference, we can separate problem $\mathsf{P_R}$ into two individual problems $\mathsf{P_D}$ and $\mathsf{P_U}$, which maximizes the downlink sum-rate and the uplink sum-rate, respectively, defined as follows:

$$(\mathsf{P_D}) \text{ maximize } \sum_{n=1}^{N} \sum_{s=1}^{S} x_{n,s} \log\left(1 + q_{n,s} d_{n,s}\right) \tag{35}$$

$$\text{subject to } \sum_{n=1}^{N} x_{n,s} \leq 1, \forall s \in \mathcal{S} \tag{36}$$

$$\sum_{n=1}^{N} \sum_{s=1}^{S} q_{n,s} \leq \mathrm{P}_{BS}, \tag{37}$$

$$q_{n,s} \geq 0, \forall n \in \mathcal{N}, \forall s \in \mathcal{S} \tag{38}$$

$$x_{n,s} \in \{0, 1\}, \forall n \in \mathcal{N}, \forall s \in \mathcal{S}. \tag{39}$$

[1]Refer to [17] for the maximum achievable subcarrier sum-rate when the uplink node and the downlink node are different.

$$(\mathsf{P_U})\ \text{maximize}\ \sum_{n=1}^{N}\sum_{s=1}^{S} y_{n,s}\log(1+p_{n,s}u_{n,s}) \tag{40}$$

$$\text{subject to}\ \sum_{n=1}^{N} y_{n,s} \le 1, \forall s \in \mathcal{S} \tag{41}$$

$$\sum_{s=1}^{S} p_{n,s} \le P_n, \forall n \in \mathcal{N} \tag{42}$$

$$p_{n,s} \ge 0, \forall n \in \mathcal{N}, \forall s \in \mathcal{S} \tag{43}$$

$$y_{n,s} \in \{0,1\}, \forall n \in \mathcal{N}, \forall s \in \mathcal{S}. \tag{44}$$

The optimal solution of $\mathsf{P_D}$ is to assign a subcarrier to a node with the largest (downlink) channel gain and to allocate some power according to the water-filling [11]. For $\mathsf{P_U}$, although no low-complexity solution has been known, its upper bound can be characterized in polynomial time [13]. To find an upper bound, we relax $\mathsf{P_U}$ to problem P'_U by replacing S constraints in (41) with a single constraint

$$\sum_{n=1}^{N}\sum_{s=1}^{S} y_{n,s} \le S. \tag{45}$$

Eq. (45) means that a subcarrier can be assigned to more than one node (e.g., $y_{n,s} = 1$ and $y_{m,s} = 1$) as long as the number of assigned subcarriers is no greater than S. Since $\{y_{n,s}\}$ satisfying (41) also satisfies (45), the optimal solution to P'_U achieves an upper bound of $\mathsf{P_U}$. From (45), if node n is allowed to use s_n subcarriers, it can now choose the best s_n subcarriers regardless of whether or not those subcarriers are used by other nodes. Then, the problem is to determine the optimal values for $\{s_n\}_{\forall n}$ satisfying $\sum_{n=1}^{N} s_n \le S$, and it can be solved using dynamic programming with a polynomial complexity in N and S [13]. Finally, by combining the optimal solution to $\mathsf{P_D}$ and the upper bound of $\mathsf{P_U}$, we can find an upper bound of $\mathsf{P_R}$ in polynomial time, which is also an upper bound of P.

VII. Numerical Results

In this section, we evaluate our resource allocation solution through numerical simulations. We use typical parameter values of LTE systems [19]: Each subcarrier has 15 kHz bandwidth and the noise power in each subcarrier is set to -130 dBm. The total transmission powers for the BS and each node n are set to 48 dBm and 24 dBm, respectively. Also, we assume zero (transmission and reception) antenna gain. For the path loss model, we use the Hata propagation

TABLE I
SIMULATION PARAMETERS

Parameter	Value
Center frequency	2.1 GHz
Subcarrier bandwidth	15 kHz
Noise power	-130 dBm
Base station's power P_{BS}	48 dBm
Node n's power P_n	24 dBm
Antenna gain	0.0
h_{BS}	30 m
h_N	1.5 m

model for urban environments where path loss P_{loss} (dB) is calculated as [12], [13], [19]

$$\begin{aligned} P_{loss}(d) &= 69.55 + 26.16 \cdot \log f - 13.83 \cdot \log h_B \\ &\quad - C_H(f) + (44.9 - 6.55 \cdot \log h_{BS}) \log d, \end{aligned} \tag{46}$$

where d (km) denotes the distance between the transmitter and the receiver, f (MHz) is the center frequency, and h_{BS} (m) denotes the height of the BS. $C_H(f)$ is the antenna height correlation factor defined as,

$$C_H(f) = 0.8 + (1.1 \cdot \log f - 0.7)\, h_N - 1.56 \cdot \log f, \tag{47}$$

where h_N (m) is the height of the terminal node. We assume that each node is placed at an equal distance from the BS, denoted as D. Table I summarizes our simulation settings.

We consider a time-slotted system. For channel model, we assume *i.i.d* Rayleigh block fading channel, where the channel gain for each subcarrier follows *i.i.d* Rayleigh distribution across users and time slots. We consider both symmetric and asymmetric channel models. In symmetric channel model, the uplink and downlink channel gains for each subcarrier are the same, and in asymmetric channel model, they are independently chosen. Note that the reciprocity-in-order also holds in the symmetric channel model.

For performance evaluation, we compare the following schemes:

- Upper Bound (UB): Performance upper bound obtained from Section VI.
- Full-Duplex Optimal solution (FD-O): The optimal solution obtained by exhaustive search. Since the complexity to find the optimal solution is $O(N^K)$, we try FD-O only for small-size

networks.

- Full-Duplex Proposed solution (FD-P): Proposed subcarrier assignment and power allocation algorithm.
- Full-Duplex Downlink optimal solution (FD-D): Assign each subcarrier to a node with the largest downlink channel gain (i.e., channel-based assignment) and allocate (uplink and downlink) power according to the water-filling. This is the optimal solution to the downlink sum-rate maximization problem P_{D}.
- Full-Duplex Uplink heuristic solution (FD-U): Assign subcarriers according to [13] and allocate (uplink and downlink) power according to the water-filling. The algorithm in [13] is a heuristic solution to the uplink sum-rate maximization problem P_{U}.
- Half-Duplex (HD): Downlink transmission and uplink transmission switch over time slots. In downlink, each subcarrier is assigned to a node with the largest downlink channel gain. In uplink, the algorithm from [13] is used for subcarrier assignment. The power is allocated by the water-filling for both downlink and uplink.

A. Symmetric channel model

We compare the empirical performance of FD-P with our analytical upper bound (UB). We set the number of nodes N to 50 and $D = 500$ m. Fig. 2 shows the performances of UB, FD-P, and HD as the number of subcarriers S increases from 10 to 100. For each value of S, the performance gap between UB and FD-P is negligible. This means that FD-P is near-optimal while UB gives a near-tight upper bound. Also, FD-P achieves almost twice as large sum-rate as HD, implying that it can fully exploit the full-duplex capability.

We also investigate how the performance gap between UB and FD-P changes according to N. As shown in Fig. 3, the sum-rate of FD-P approaches even closer to the upper bound with a larger N. The performance gap is 1.7% for $N = 10$, and decreases to 0.3% for $N = 200$. As the number of nodes increases, our solution converges to the channel-based subcarrier assignment, which is optimal for an infinite number of nodes. Therefore, the performance of our solution becomes closer to the upper bound for a larger number of nodes.

We calculate the probability P_{all+} of $(\hat{P}, \hat{Q})$ being an all-positive power allocation vector. Note that P_{all+} is a lower bound of the probability that our solution guarantees local Pareto optimality. Fig. 4 shows that for each value of S, P_{all+} approaches 1 very quickly even with

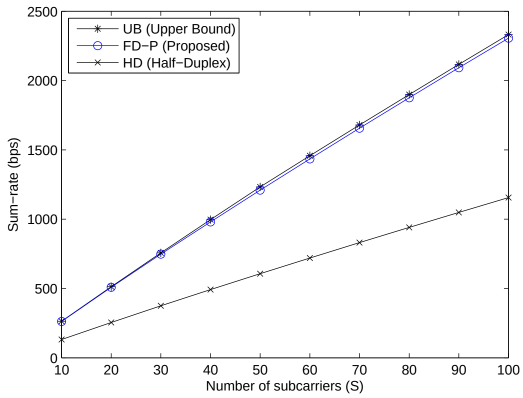


Fig. 2. Performance of UB, PD-P, and HD in symmetric channel when $N = 50$ and $D = 500$ m. The sum-rate of each scheme linearly increases with the number of subcarriers S due to the growing bandwidth. The performance gap between UB and FD-P is negligible, showing that FD-P achieves near-optimal performance.

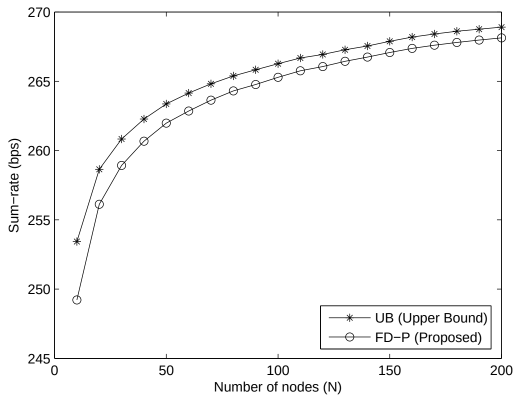


Fig. 3. Performance comparison between UB and FD-P when $D = 500$ m and $S = 10$. The performance gap diminishes as N increases from 10 to 200, and it becomes 1.7% for $N = 10$ and 0.3% for $N = 200$.

a small number of nodes, e.g., less than 10 nodes. For example, when $S = 50$, P_{all+} reaches 1 only after N becomes 9. Due to the node diversity, each subcarrier is assigned to a node with a larger channel gain when there are more nodes. As a result, each subcarrier is assigned positive (uplink and/or downlink) power with high probability. Note that in most modern cellular systems, the number of nodes in a cell is typically larger than 10.

B. Asymmetric Channel Case

We evaluate FD-P in comparison with UB and FD-O in small-size networks. As shown in Fig. 5, FD-P achieves almost the same performance as FD-O, and thus FD-P is empirically

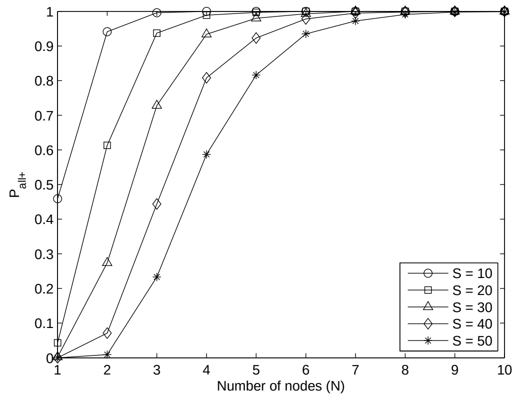


Fig. 4. The probability P_{all+} of having an all-positive power allocation vector for $D = 500$ m. For each value of S, P_{all+} sharply converges to 1 even before N reaches 10.

near-optimal even in asymmetric channel environments. The performance gap between UB and FD-P ranges from 8% to 9%, which is a huge increase compared to the symmetric channel case. In asymmetric channel environments, the full-duplex optimal solution is close neither to the uplink optimal solution nor to the downlink optimal solution. As a result, the analysis based on separation of uplink and downlink transmissions results in a loose upper bound.

We now compare the performance of each scheme in large-size networks. We omit FD-O due to its high computational complexity and include UB and HD as an upper bound and a lower bound, respectively. Fig. 6 shows that FD-P outperforms both FD-D and FD-U substantially. The performance gain of FD-P over FD-D and FD-U increases with the number of subcarriers. As S increases from 10 to 100, FD-P shows the gain of 9.7% to 11.1% over FD-D and the gain of 13.6% to 17.4% over FD-U.

We also investigate how the performance of each scheme varies with N. Fig. 7 shows the sum-rate of each scheme as N increases from 10 to 100. When $N = 10$, the performance gap between FD-P and FD-D is negligible. Since P_{BS} $(= 48$ dBm$)$ is about 250 times larger than P_n $(= 24$ dBm$)$, the downlink sum-rate dominates the full-duplex sum-rate when there are few nodes. This explains why FD-P and FD-D show similar performance in case of small N. As N grows, however, the sum of each node's uplink power, i.e., $N \cdot P_n$, also linearly increases, balancing the impact of downlink and uplink on the full-duplex sum-rate. As a result, the performances of FD-D and FD-U become closer while FD-P outperforms both.

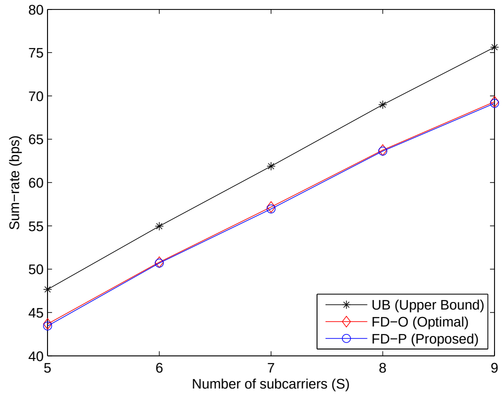


Fig. 5. Performance comparison in small-size networks where $N = 5$ and $D = 500$ m. FD-P achieves almost the same performance with FD-O, and thus it is empirically near-optimal. The upper bound is not tight, and shows a gap of 11.2% to 16.9% from FD-P.

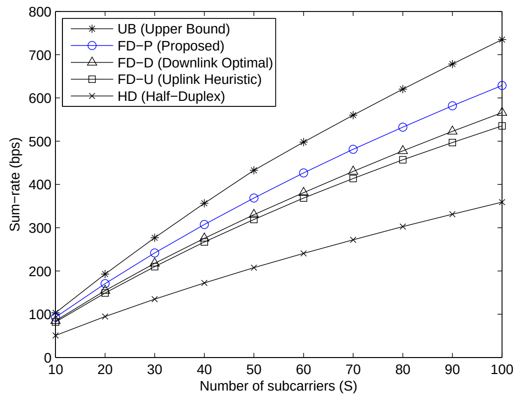


Fig. 6. Performance comparison in large-size networks when $N = 50$ and $D = 500$ m. FD-P outperforms FD-D and FD-U with a gain of 9.7% to 11.1% and 13.6% to 17.4%, respectively.

We investigate the impact of the node-exclusive subcarrier assignment on the performance by comparing the performance of our FD-P and HD. Since HD can assign each subcarrier to different (uplilnk and downlink) users and thus fully exploit node diversity, its performance substantially improves as N increases as shown in Fig. 8(b). Under FD-P, however, the node-exclusive subcarrier assignment restricts node diversity, and thus its performance improvement is less remarkable than HD as shown in Fig. 8(a). Note that we scale the y-axis of both Figs. 8(b) and 8(a) to show up to 2.5 times the minimum achieved sum-rate.

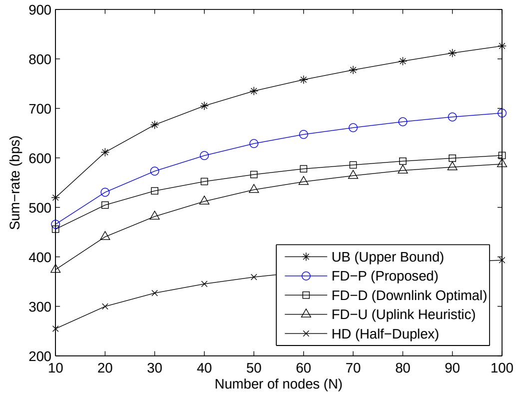
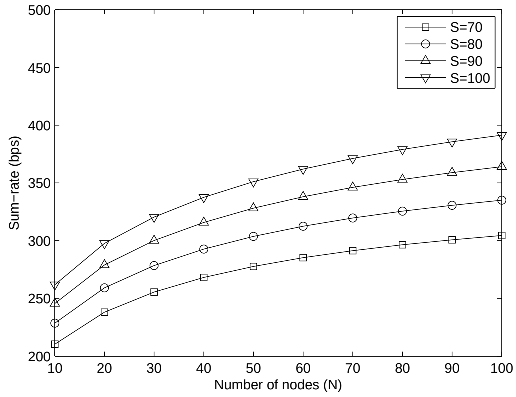
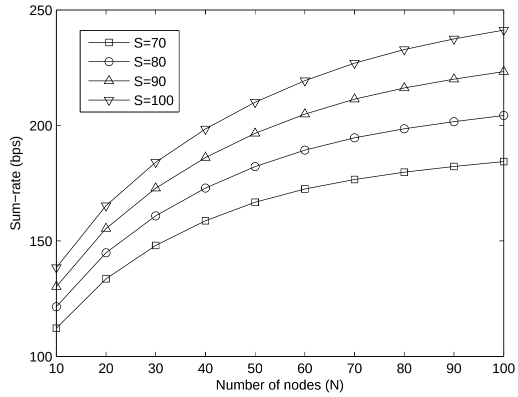


Fig. 7. Performance comparison for different values of N. For small N, FD-D shows a similar performance to FD-P due to the dominant impact of downlink sum-rate on the full-duplex sum-rate. As N increases, the performances of FD-D and FD-U become similar while FD-P outperforms both of them.



(a) Sum-rate of FD-P



(b) Sum-rate of HD

Fig. 8. Performance comparison of FD-P and HD. Since the node-exclusive subcarrier assignment restricts node diversity, the sum-rate of FD-P grows less rapidly than that of HD.

VIII. Related Work

Full-Duplex Implementation: Self-interference cancellation techniques can be categorized into antenna cancellation, analog cancellation, and digital cancellation. An antenna cancellation technique was first proposed in [2] where a pair of transmission antennas are placed such that the signal from one antenna cancels out that from the other. For a wavelength λ, two transmission antennas are placed at d and $d + \frac{\lambda}{2}$ away from the reception antenna to make their signals add destructively. Analog cancellation uses the known transmission signal to cancel out the self-interference in RF signal domain. The received signal is added with an inverted copy of transmitted analog signal which is generated by a second transmit chain [20] or a special component such as balun transformer [21]. Digital cancellation is used to clean out any remaining self-interference which is generated by non-ideal and non-linear components in an RF chain. The state-of-the-art work has demonstrated that full-duplex can be implemented with a single antenna, covering up to 80 MHz of bandwidth [6]. This can be realized by a hybrid analog-digital cancellation technique that accurately models all linear and non-linear distortions of signals in a TX chain.

Full-Duplex Power Control: The power allocation for full-duplex communication has been recently studied in the literature. An optimal power control scheme for the sum-rate maximization of two-node bidirectional transmissions has been proposed in [22]. The authors showed that the sum-rate maximization problem can be converted into a convex optimization problem and its solution can be calculated numerically. An optimal power allocation scheme for three-node relay transmission has been developed in [23]. In [17], the authors characterized the achievable rate region in a three-node full-duplex network with a side channel. They also showed how inter-node interference can be mitigated with the help of an orthogonal side-channel between uplink and downlink nodes. An important limitation of previous works is that they have not considered multiuser environments where subcarrier assignment should be jointly considered with power allocation.

Resoure Allocation in OFDMA: Resource allocation for sum-rate maximization in downlink or uplink OFDMA systems has been extensively studied. In a downlink case, the optimal solution is to assign each subcarrier to a node with the best channel condition and to allocate some power to it according to the water-filling method [11]. In an uplink case, several suboptimal algorithms have been proposed. In [12], the authors proposed two greedy algorithms which are motivated by a necessary condition for optimality. In [13], a necessary condition for optimality is extended to cover a general utility function, and a low-complexity allocation algorithm is proposed. In [14], the authors changed the original problem into a convex optimization problem by relaxing exclusive subcarrier assignment and found an optimal solution to the relaxed problem. The optimal solution was then used to guide several heuristic algorithms for the original problem. A detailed survey for downlink and uplink is presented in [15] and [16], respectively.

IX. Conclusion

Full-duplex transmission is a promising technology to boost the network capacity. While a near-double capacity is anticipated in point-to-point wireless links, the performance gain remains unclear in multiuser and multicarrier networks where radio resource should be optimally allocated to each node. In this paper, we have developed a new radio resource allocation algorithm for full-duplex OFDMA networks using a necessary condition for the optimal solution. The proposed algorithm assigns subcarriers to nodes in an iterative manner with low complexity. We prove that our algorithm achieves local Pareto optimality under certain conditions that hold frequently in practice. By separating the uplink and downlink transmissions, we have obtained an upper bound on performance that is near-tight in symmetric channel cases. Through extensive numerical simulations, we demonstrate that our algorithm achieves near-optimal performance and outperforms other resource allocation schemes designed for half-duplex networks.

There remain many interesting resource allocation problems in full-duplex networks. For example, the tradeoff between node diversity and channel feedback overhead for inter-node links needs further investigation. Although there are many interesting results on efficient channel feedback methods [24], [25], [26], they are limited to channel gains between BS and each node. This means that the benefit and cost of inter-node channel gains are not fully understood yet. Another interesting open problem is the scheduling in multicell environments, where each downlink node can suffer from interference coming from uplink nodes in neighboring cells.

Appendix A
Proof of Lemma 1

Given a channel gain vector $G = \{u_{n,s}, d_{n,s}\}$, let X denote a feasible subcarrier assignment and (P^X, Q^X) denote the corresponding (uplink and downlink) power allocation by the water-filling. We assume that (i) G has elementwise-unique channel gains and satisfies the reciprocity-in-order and (ii) (P^X, Q^X) is an all-positive power allocation vector. Let P_n^X and Q_n^X denote the uplink and downlink powers of node n, respectively, i.e., $P_n^X = \sum_s p_{n,s}^X$ and $Q_n^X = \sum_s q_{n,s}^X$. Also, let U_n^X and D_n^X denote the uplink sum-rate and the downlink sum-rate of node n, respectively, defined as

$$U_n^X = \sum_{s \in S_n^X} \log\left(1 + p_{n,s}^X u_{n,s}\right), \tag{48}$$

$$D_n^X = \sum_{s \in S_n^X} \log\left(1 + q_{n,s}^X d_{n,s}\right). \tag{49}$$

Then we have $R_n^X = U_n^X + D_n^X$.

Now we provide Lemma 3 and Lemma 4 to prove Lemma 1 as follows:

Lemma 3. Suppose that subcarrier assignment is changed from X to $Y \in B(X, 1)$. If there exists a node which only loses a subcarrier and wins no subcarrier, then Y does not Pareto dominate X.

Proof: We prove this by contradiction. Suppose Y Pareto dominates X. Without loss of generality, assume that there exists only one node which loses a subcarrier. Suppose that node n loses subcarrier s, i.e., $\mathcal{S}_n^Y = \mathcal{S}_n^X \backslash \{s\}$. Since Y Pareto dominates X, we have $R_n^Y \geq R_n^X$. From the assumption that (P^X, Q^X) is an all-positive power allocation vector, we have $p_{n,s}^X + q_{n,s}^X > 0$. Now, we can consider the following two cases.

1) For $p_{n,s}^X > 0$:

   Since node n loses subcarrier s with positive uplink power, $U_n^Y < U_n^X$ must be true considering the characteristic of the water-filling. The reduced uplink sum-rate should be compensated by the increase in the downlink sum-rate, i.e., $D_n^Y > D_n^X$, which implies $Q_n^Y > Q_n^X$.

2) For $q_{n,s}^X > 0$:

   From $U_n^Y \leq U_n^X$, we should have $D_n^Y \geq D_n^X$. Since node n loses subcarrier s with positive downlink power, more downlink power should be allocated in Y than in X, i.e., $Q_n^Y > Q_n^X$.

In both cases, we should have $Q_n^Y > Q_n^X$, which means that (i) the downlink water level α^Y in Y is greater than the downlink water level α^X in X, and (ii) there exists at least one node which loses some of its downlink power, part of which is reallocated to node n. Now consider a node m which obtains subcarrier l, i.e., $\mathcal{S}_m^Y = \mathcal{S}_m^X \cup \{l\}$. Since $\alpha^Y > \alpha^X$, we can prove that

$Q_m^Y \geq Q_m^X$ as follows:

$$\begin{aligned} Q_m^Y &= \sum_{w \in \mathcal{S}_m^Y} \left[\alpha^Y - \frac{1}{d_{m,w}} \right]^+ \\ &= \sum_{w \in \mathcal{S}_m^X} \left[\alpha^Y - \frac{1}{d_{m,w}} \right]^+ + \left[\alpha^Y - \frac{1}{d_{m,l}} \right]^+ \\ &\geq \sum_{w \in \mathcal{S}_m^X} \left[\alpha^Y - \frac{1}{d_{m,w}} \right]^+ \\ &\geq \sum_{w \in \mathcal{S}_m^X} \left[\alpha^X - \frac{1}{d_{m,w}} \right]^+ = Q_m^X. \end{aligned} \tag{50}$$

Next, consider node m which swaps subcarrier i for subcarrier j, i.e., $\mathcal{S}_m^X \backslash \{i\} = \mathcal{S}_m^Y \backslash \{j\}$. If $d_{m,i} < d_{m,j}$ (and also $u_{m,i} < u_{m,j}$ by the reciprocity-in-order), Q_m^Y should be larger than Q_m^X to have $R_m^Y \geq R_m^X$. If $d_{m,i} > d_{m,j}$, we have $Q_m^Y \geq Q_m^X$, which can be proven as

$$\begin{aligned} Q_m^Y &= \sum_{w \in S_m^Y} \left[\alpha^Y - \frac{1}{d_{m,w}} \right]^+ \\ &= \sum_{w \in S_m^Y \backslash \{j\}} \left[\alpha^Y - \frac{1}{d_{m,w}} \right]^+ + \left[\alpha^Y - \frac{1}{d_{m,j}} \right]^+ \\ &= \sum_{w \in S_m^X \backslash \{i\}} \left[\alpha^Y - \frac{1}{d_{m,w}} \right]^+ + \left[\alpha^Y - \frac{1}{d_{m,j}} \right]^+ \\ &\geq \sum_{w \in S_m^X \backslash \{i\}} \left[\alpha^X - \frac{1}{d_{m,w}} \right]^+ + \left[\alpha^X - \frac{1}{d_{m,j}} \right]^+ \\ &\geq \sum_{w \in S_m^X \backslash \{i\}} \left[\alpha^X - \frac{1}{d_{m,w}} \right]^+ + \left[\alpha^X - \frac{1}{d_{m,i}} \right]^+ \\ &\geq \sum_{w \in S_m^X} \left[\alpha^X - \frac{1}{d_{m,w}} \right]^+ = Q_m^X. \end{aligned} \tag{51}$$

Considering the above cases, there exists no node which is allocated less downlink power in Y than in X. As a result, $Q_n^Y > Q_n^X$ does not hold. This means that we have $R_n^Y < R_n^X$, which contradicts that Y Pareto dominates X. ■

Lemma 4. Assume that each node swaps a subcarrier for another one or maintains its subcarrier assignment. If there exists a node which swaps a subcarrier for another one with a smaller (uplink and downlink) channel gain, then Y does not Pareto dominate X.

Proof: Again, we prove this by contradiction. Without loss of generality, assume that node n swaps subcarrier s for subcarrier l with $d_{n,s} < d_{n,l}$ and $u_{n,s} < u_{n,l}$, while each of the other nodes swaps one of its subcarriers for another one with a larger channel gain. Since $d_{n,l} > d_{n,s}$ and $p_{n,s} + q_{n,s} > 0$, $Q_n^Y = Q_n^X$ leads to $R_n^Y < R_n^X$. Thus we should have $Q_n^Y > Q_n^X$, which means that there exists a node which is allocated less downlink power in Y than in X. However, we can prove by (51) that there exists no such node, which is a contradiction. Thus, Y does not Pareto dominate X. ∎

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