

# Energy Efficient Greedy Link Scheduling and Power Control in wireless networks

\*Arun Sridharan, †Changhee Joo, and \*C. Emre Koksal

\*Department of Electrical and Computer Engineering  
The Ohio State University  
Columbus, Ohio 43210  
{sridhara,koksal}@ece.osu.edu

†School of Electrical and Computer Engineering  
Ulsan National Institute of Science and Technology  
Ulsan, South Korea 330-708  
cjoo@unist.ac.kr

**Abstract**—We consider the problem of joint link scheduling and power control for wireless networks with average transmission power constraints. Due to the high computational complexity of the optimal policies, we extend the class of greedy link scheduling policies to handle average power constraints. We develop a greedy link scheduling and power control scheme GECS, with provable performance guarantees. We show that the performance of our greedy scheduler can be characterized using the Local Pooling Factor (LPF) of a network graph, which has been previously used to characterize the stability of the Greedy Maximal Scheduling (GMS) policy for wireless networks. We also simulate the performance of GECS on wireless network, and compare its performance to another candidate greedy link scheduling and power control policy.

## I. INTRODUCTION

Energy efficient networking principles have received significant attention in the recent past. Network devices are often provisioned with multiple transmit power levels, but are limited by peak as well as average energy constraints. These constraints are determined by factors such as amplifier characteristics, energy costs and battery limitations. Consequently, one needs efficient control strategies aimed at achieving different performance objectives such as maximizing network lifetime, or maximizing network throughput while satisfying the energy constraints for the network. Conversely, in many scenarios, one wants to guarantee a certain network throughput region, while minimizing long term average energy expended to sustain any admissible arrival rate. In this paper, we consider the problem of link scheduling over a wireless network in the presence of average energy constraints. Owing to the high computational complexity of the optimal policy, we propose low complexity energy efficient link scheduling and power control policies. We exploit the property of Longest Queue First (LQF)-based algorithms to develop a low-complexity greedy power control and scheduling scheme (GECS) for wireless networks under binary interference models. We then analyze its performance in wireless networks and show that their performance can be characterized using the LPF Of a wireless network. We thus show that the LPF can be used to characterize the performance of cross-layer schemes, spanning multiple layers.

A link scheduler selects a non-interfering subset of links in a wireless network to schedule at every instance. A throughput optimal link scheduler is one that keeps the queues in the network stable for the largest set of admissible arrival rates for that network. However, when constraints on average

transmission power are imposed, a link scheduler could end up expending more average power than necessary, even when the arrival rate is admissible, as seen from the following example: Consider the case of transmission over a single link where one is allowed to either transmit 2 packets in a time slot by expending 2 units of power, or 1 packet in a time slot by expending 0.75 units of power. Suppose the arrival process is such that two packets arrive at the start of every odd numbered time slot. The link scheduler would cause the link to transmit whenever there are packets in the queue, *i.e.*, every other time slot, consequently requiring an average power of 1 unit. If the average power constraint was less than one units, the link scheduler would be unable to meet the power constraint, whereas by transmitting one packet in each time slot one would consume a lesser average power of 0.75 units and yet guarantee the same throughput. The same limitations of the link scheduler carries over to the scenario in which multiple interfering links are present. Thus, link scheduling needs to be combined with an effective power control strategy in order to ensure that the largest set of admissible arrival rates can be stabilized while meeting average energy requirements. The optimal link scheduling and power control policy, proposed in [3] however, suffers from high computation complexity, which is also one of the main challenges in optimal link scheduling. It is thus natural to ask if one can design low-complexity link scheduling and power control policies, motivated by the study of low-complexity link schedulers. In this context, our contribution is that we extend the scope of greedy schedulers such as Longest-Queue-First and Greedy Maximal Scheduling [10] to handle energy constraints in wireless networks. Our link scheduling and power control policy, GECS, is based on properties of both greedy schedulers as well as optimal schedulers. Interestingly, we are able to characterize the performance of GECS in terms of the LPF of a network graph, a parameter that has been used to characterize the performance of GMS in wireless networks.

Previous studies have focused on energy efficiency in a particular aspect of network control, (e.g., Energy efficient routing, or energy efficient MAC protocols) as well as over control decisions spread across the network layers; For instance, [1] and [2] consider the problem of joint routing, scheduling and power control in wireless networks. In [1], the authors develop a power control and link scheduling policy

that minimizes total average power, while satisfying minimum rate constraints over each link. The optimal policy has high computational complexity, exponential in general. The model moreover does not consider the stochasticity of arrivals and dynamic queue control. In [2], the authors provide a low complexity approximation algorithm to solve the joint routing, scheduling and power control problem in wireless networks under a node-exclusive interference model.

While the above network optimization problems assume that the long term average rate constraints are known, this is not true in general. Stochastic network optimization considers the problem of dynamic queue control in the presence of unknown arrival rates in order to optimize some quantity such as total average expended power. Stochastic network optimization problems have been addressed using stochastic gradient techniques ([4], [6]) or Lyapunov optimization techniques [3]. In [3], a Lyapunov drift based optimization technique was used to obtain asymptotically optimal policies that minimized the total average power for any arrival rate feasible under the peak power constraints. Lyapunov based optimization has also been used in energy constrained scheduling, where the objective is to maximize the network throughput region (set of stabilizable arrival rates) subject to average energy constraints. Subsequently, such techniques have been used in dynamic channel acquisition and scheduling to minimize energy ([3], [5]) consumed for transmission as well as channel acquisition, or to maximize throughput when only limited channel information may be acquired by a link. In all these studies however, the optimal policy has high computational complexity motivating the study of low complexity algorithms with good performance guarantees. While low complexity approximation algorithms have been proposed in specific interference models [7], or for specific networks [6], finding a low complexity dynamic control algorithm with good performance guarantees remains largely unaddressed.

In the context of link scheduling, the LQF policy, and its variant Greedy Maximal Scheduling (GMS) have been analyzed extensively as low-complexity scheduling policy for wireless networks. The performance of GMS has been characterized recently using the Local Pooling Factor, a graph theoretic parameter, and LPF has been studied for a large class of graphs and interference models [10], [15]. It is thus beneficial to integrate power control with existing greedy link scheduling policies. In the next section, we describe the system model. We then describe and analyze the GECS policy in Sections II and III respectively.

## II. SYSTEM MODEL

We consider a wireless network represented by a network graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ . For simplicity, we initially consider wireless networks with single-hop traffic, so that each link  $l$  is a source-destination pair. We assume that link  $l$  is associated with interference set  $I_l$ , so that if link  $l$  is active, then no other link in  $I_l$  can be active. Every link  $l$  is associated with a peak transmit power constraint. We also assume that a transmitter can transmit using a discrete set of power levels  $\{P_l^1, P_l^2, \dots, P_l^{max}\}$ . These discrete power levels are chosen

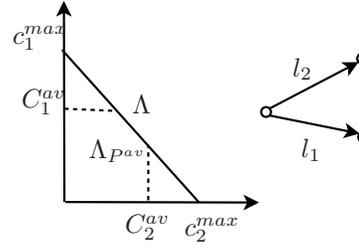


Figure 1: Stability region for a two link network graph.  $C_1^{av}$  and  $C_2^{av}$  represent the maximum admissible arrival rate for the two links. The stability region  $\Lambda_{P^{av}}$  is the region enclosed within dotted lines.

from a convex rate-power curve represented by a function  $f_l(c_l)$ , where  $f_l(c_l)$  is the power required to transmit at rate  $c_l$ . The function  $f_l(c_l)$  depends on the channel conditions of link  $l$ , which are assumed to be static for the wireless network. For example, in the case of an AWGN channel with a bandwidth  $W$ , noise spectral density  $N_0$ , and a power gain  $h_l$ , the power consumed for a rate  $c_l$  is given by  $\frac{N_0 W}{h_l} \left(2^{\frac{c_l}{W}} - 1\right)$  (Assuming Gaussian codewords with average power  $P_l$ ). The rates corresponding to the available transmit power levels are denoted by  $\{c_l^1, c_l^2, \dots, c_l^{max}\}$ .

We further assume that each link is also associated with an average power constraint  $P_l^{av}$ , so that  $\limsup_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T P_l(t) \leq P_l^{av}$ . We assume that time is slotted so that  $P_l(t)$  is the power consumed during time slot  $t$ . Let  $Q_l$  denote the queue of link  $l$  into which packets arrive at the beginning of every time slot. Packets arrive into queue  $Q_l$  according to an I.I.D. process with finite mean  $\lambda_l$ .

For a given vector of transmit powers  $\vec{P}(t)$ , let  $\vec{r}, \{r_l = f_l^{-1}(P_l)\}$  denote the rate allocation vector achieved by using  $\vec{P}(t)$  in time slot  $t$ . We define a rate allocation vector  $\vec{r}$  as being feasible if it satisfies interference constraints, i.e.,  $r(l) \neq 0 \Rightarrow r(k) = 0, \forall k \in I_l, k \neq l$ . Let  $R = \{\vec{r} | \vec{r} \text{ is feasible}\}$  and  $P = \{\vec{P} | \vec{r} \text{ defined by } r_l = f_l^{-1}(P_l) \text{ is feasible}\}$  denote the set of all feasible rate allocation vectors and power allocation vectors for the network. A scheduling policy  $\pi : Q \rightarrow P$  selects a feasible power allocation vector in each time slot  $t$ .

Let  $\vec{P}^{av}$  denotes the vector of average power constraints for all links. The power constrained stability region  $\Lambda_{P^{av}}$  can be then defined as [3], [16]:

$$\Lambda_{P^{av}} = \{\vec{\lambda} | \exists \vec{P}(1), \vec{P}(2), \dots \text{ such that} \quad (1)$$

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \vec{r}(t) \succ \vec{\lambda} \text{ and } \vec{P}^{av} \succeq \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \vec{P}(t)\},$$

where  $\succ$  and  $\succeq$  represent component wise inequalities. Fig. 1 shows  $\Lambda_{P^{av}}$  for a simple two link network graph with transmit power levels  $\mathcal{P}_l = \{0, P_l^{max}\} l = 1, 2$  and average power constraints  $P_l^{av} < P_l^{max}$ . Links  $l_1$  and  $l_2$  are in each other's interference sets.  $C_l^{av}$  denotes the highest admissible arrival rate for link  $l$ . Note that when the available transmit power levels for each link  $l$  is increased to  $\hat{\mathcal{P}}_l = \{0, P_l^1, P_l^{max}\} \supset \mathcal{P}_l, l = 1, 2$ , then the power constrained stability region increases as well, so that  $\Lambda_{P^{av}, \hat{\mathcal{P}}} \supseteq \Lambda_{P^{av}, \mathcal{P}}$ . This follows from

the convexity of rate-power curve  $f_l(c_l)$ . When the average power constraint  $P_l^{av}$  is set to  $P_l^{max}$ , then every link  $l$  could always transmit at maximum power  $P_l^{max}$  when scheduled, and no power control would be necessary. Otherwise, when  $P_l^{max} > P^{av}$ , then a joint link scheduling and power control policy is required even when the link channel conditions are static. We are interested in a joint power control and link scheduling policy that stabilizes the queues in the network for any arrival rate in  $\Lambda_{P^{av}}$  while satisfying the power constraints.

In order to satisfy the average power constraints, we employ virtual power queues for each link, as first proposed in [3]. Every link  $l$  keeps a virtual queue  $U_l$  that is used to track the excess power expended beyond the link's power constraint  $P_l^{av}$ . The departures from the virtual queue  $U_l$  follow an I.I.D. process with mean  $P_l^{av}$ . The arrivals in each time slot represent the power expended during that slot. Let  $P_l(t)$  be the power which link  $l$  transmits during time slot  $t$ , then  $P_l$  packets are assumed to have arrived at the beginning of time slot  $t + 1$ . The dynamics of the real and virtual queues in each time slot are then given by:

$$U_l(t+1) = [U_l(t) - P_l^{av}(t)]^+ + P_l(t), \text{ and} \\ Q_l(t+1) = [Q_l(t) - S_l(t)]^+ + A_l(t).$$

From Lemma 3 of [3], if the virtual queue  $U_l$  is stable, then the average power constraint is satisfied. An optimal scheduling policy is one which can stabilize any  $\vec{\lambda} \in \Lambda_{P^{av}}$ , while ensuring that the power constraints in each link are satisfied. An optimal scheduling strategy is given in [3] as:

$$\operatorname{argmax}_{P_l, l \in \mathcal{E}} \sum_{l \in \mathcal{E}} (Q_l(t)r_l(\vec{P}(t)) - U_l(t)P_l(t)) \quad (2)$$

The computational complexity of the optimal policy is however, very high, formidably hard with most interference models [9]. Indeed, even under the binary interference model, eq. (2) describes a Max-weight policy as the solution maximizes the sum of weights of each link in the selected transmission power vector, where the weight of link  $l$  is given by  $\max_{P_l} Q_l f^{-1}(P_l) - U_l P_l$ . The Max-weight scheduler is known to be NP-hard in many cases including K-hop interference models [9]. This motivates the need for a low complexity power control and link scheduling policy with good performance guarantees.

In the following section, we describe our link scheduling and power control policy, the Greedy Energy Constrained Scheduler (GECS).

### III. GREEDY ENERGY CONSTRAINED SCHEDULER (GECS)

The GECS scheduler exploits the properties of both LQF as well as Maxweight policies in order to perform link scheduling and power control. The GECS scheduler first greedily selects links, based on the queue weights  $Q_l^2 + U_l^2$ . It then assigns power levels optimally to the selected link, by applying the optimal power control strategy in (2) as if link  $l$  alone is active. This procedure continues until no more links remain to be selected. The GECS scheduler can be formally described as follows: We first initialize the set  $Z$  to  $\mathcal{E}$ .

- (1) Select link  $m \leftarrow \operatorname{argmax}_{l \in Z} \{Q_l^2 + U_l^2\}$ .

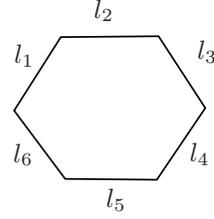


Figure 2: Six-cycle network graph

- (2) Select the power level as:
$$P_m^* \in \operatorname{argmax}_{P_m \in \{P_m^1, P_m^2, \dots, P_m^{max}\}} Q_m f_m^{-1}(P_m) - U_m P_m.$$
- (3a) The scheduler then sets  $r(m) = f_m^{-1}(P_m^*)$ , and  $r(k) = 0$ , for all  $k \in I_m \cap Z$ , with  $k \neq m$ .
- (3b) Update  $Z$  as  $Z \leftarrow Z / I_m$  to remove interfering links.
- (4) If  $Z \neq \{\emptyset\}$ , then repeat step 1.

In step 1 as well as step 3, the GECS policy breaks ties randomly. Note that step 2 is only performed on links selected in step 1, and does not increase the computational complexity of the greedy policy, which is  $O(|\mathcal{E}| \log |\mathcal{E}|)$  considering a centralized implementation. The following example illustrates the GECS policy on a 6-cycle network graph under the one-hop interference model.

**Example 1.** Consider the six cycle graph shown in Fig. 2 with links  $l_1$  to  $l_6$ . Let the available transmit power levels for link  $l$  be  $P_l \in [0, 1]$ . Let the corresponding rates achieved by  $c_l \in \{0, 1\}$ . Suppose at time  $t$ , the  $Q(t) = [2 \ 3 \ 8 \ 5 \ 2 \ 10]$ , and  $U(t) = [1 \ 4 \ 7 \ 5 \ 3 \ 12]$ . Since link  $l_6$  has the highest weight  $Q_l^2 + U_l^2$ , GECS selects link  $l_6$  first. However, in step 3, the transmit power level that maximizes the weight of link  $l_6$  is 0. Thus  $l_6$  is not scheduled. The next highest weight belongs to  $l_3$  and link  $l_3$  is added to the schedule. Under the one-hop interference model, links  $l_2$  and  $l_4$  are interfering links and can not be scheduled. The scheduler then deactivates link  $l_5$ , and schedules link  $l_1$ . At the end of this procedure, the rate allocation vector is  $\vec{r} = [1 \ 0 \ 1 \ 0 \ 0 \ 0]$ . Note that GECS does not necessarily yield a schedule that is a maximal matching on the graph  $\mathcal{G}$ .

The intuition behind the performance of the GECS scheduler can be explained using the properties of the LQF and the optimal policies. For any arrival rate inside  $\Lambda_{P^{av}}$ , the optimal policy causes the drift of a quadratic Lyapunov function to be negative; LQF policy, on the other hand causes the set of longest queues to decrease in the fluid limit if the local pooling conditions are satisfied [8]. Since GECS combines LQF (for link selection) and the optimal policy (for power control), one expects GECS to cause the drift of a quadratic function to be negative whenever certain local pooling conditions are satisfied. This indeed turns out to be the case, although local pooling occurs in a manner different from [8]. Nevertheless, the local pooling enables characterizing the performance of GECS using the LQF. In the next section, we analyze the performance of GECS and make these relations precise.

### IV. PERFORMANCE CHARACTERIZATION

We analyze the performance of the GECS scheduler by characterizing its efficiency ratio. The efficiency ratio of GECS is defined as the largest fraction of  $\Lambda_{P^{av}}$ , that can be stabilized

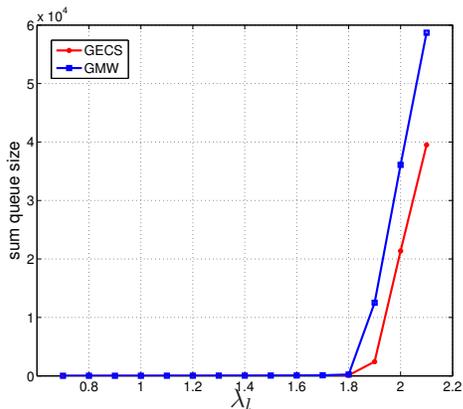


Figure 3: The performance of GECS and GMW is plotted for the network graph in Fig. 2

by GECS. We show that the efficiency ratio of GECS is can be related to the LPF of a network graph, which depends on the topology of the network and the interference model. The LPF can be defined using the local pooling conditions which are given below [11]:

**Definition 2.** Let  $L \subset \mathcal{E}$  be any subgraph of  $\mathcal{G}$ . Let  $\vec{s}_L$ , where  $s_L(l) \in \{0, 1\}, \forall l \in L$  be any  $1 \times |L|$  link activation vector with the following properties: (i)  $\vec{s}_L$  satisfies interference constraints, and (ii)  $\vec{s}_L$  is maximal, i.e.,  $\exists$  no  $l \in L$  with  $s_L(l) = 0$  such that  $l$  satisfies interference constraints. Let  $\mathcal{M}_L$  be the set of all such link activation vectors  $\vec{s}_L$ . Then,  $\mathcal{L}$  satisfies  $\sigma$ -local pooling if, for any given pair  $\vec{\mu}, \vec{\nu}$ , where  $\vec{\mu}$  and  $\vec{\nu}$  are convex combinations of elements in  $\mathcal{M}_L$ , we have  $\sigma \vec{\mu} \not\prec \vec{\nu}$ . The LPF  $\sigma^*$ , for the network is then defined as:

$$\sigma^* = \sup \{ \sigma \mid \forall \mathcal{L} \subset \mathcal{G}, \mathcal{L} \text{ satisfies } \sigma\text{-local pooling} \}.$$

In Theorem 3, we show that the efficiency ratio of the GECS policy for a network graph  $\mathcal{G}$  is lower bounded by the LPF  $\sigma^*$  of  $\mathcal{G}$ .

**Theorem 3.** Consider a network graph  $\mathcal{G}$  with LPF  $\sigma^*$ . The GECS policy stabilizes any  $\vec{\lambda} \in \sigma^* \Lambda_{P_{av}}$ .

The proof is given in Appendix A. Although GECS is also in the spirit of the LQF policy, we use a Lyapunov function  $V(Q, U) = \max_{l \in \mathcal{E}} Q_l^2 + U_l^2$ , that is different from the ones used for LQF and GMS in [8], [10]. The performance guarantees of GECS in terms of LPF is attractive as the LPF has been extensively studied in context of characterizing the performance of GMS in wireless networks [10], [11] and [14]. Indeed, [15], for instance identifies all network graphs whose LPF is 1 under node-exclusive interference models. Here we show that the LPF can be used to characterize the performance of a joint power controller and scheduler, rather than only the scheduler. This is a step towards a complete performance characterization of cross-layer schemes with a single parameter.

## V. SIMULATIONS

In this section we simulate the performance of GECS for the six-cycle graph in Fig. 2. For the same network graph, we also simulate the performance of another candidate greedy scheduler, which we call Greedy Max-weight (GMW). GMW is a greedy approximation of the optimal policy in (2) that gives priority to the link with the highest weight

$\text{argmax}_{l \in \mathcal{E}} \{ \max_{P_l} (Q_l f^{-1}(P_l) - U_l P_l) \}$ . It then proceeds by removing interfering links and repeating this process until no other interfering links remain to be scheduled. Fig. 3 plots the total queue sizes as the offered load  $\lambda_l$  in each link is increased towards its maximum admissible arrival rate. The available transmit power levels for each link are assumed to be in the set  $\{0, 1, 3, 7, 15\}$  units. We assume the node exclusive interference model. The average power constraint for each link is set at 2.75. The corresponding rates are chosen by assuming an AWGN channel so that  $C_l = \log(1 + P_l h_l / N_0)$  where the path loss  $h_l$  is arbitrarily chosen for each link but fixed over the scheduling time-frame. The highest admissible arrival rate for each link is given by  $[2 \ 3.5 \ 4.2 \ 5.1 \ 4.7 \ 4]$ . Since the LPF of the network graph is  $2/3$ , we expect GECS to simultaneously schedule one third the maximum admissible arrival rate on each link  $l$  (since by scheduling 3 of the 6 links at a time, half the maximum admissible arrival rate is admissible for every link) this is indeed seen to be the case. Although performance guarantees for the GMW policy have not been analyzed using the LPF as we did for GECS in the previous section, the plots show that GECS performs better than GMW at higher loads, and reduces the average sum queue size by 30-75%. This could be attributed to the fact that GMW only looks at the difference in queue weights and may be insensitive to links that have long queue lengths but with relatively low weight values.

## VI. CONCLUSION

In this paper, we addressed the problem of link scheduling and power control for wireless networks under average power constraints. Owing to the high computational complexity of the optimal policy, we extended the scope of greedy link schedulers to handle average power constraints for wireless networks under a binary interference model. We proposed a greedy link scheduling and power control scheme, GECS, and show that the efficiency ratio of GECS for any wireless network is lower bounded by the Local Pooling Factor (LPF) of the network graph. LPF has been previously used to characterize the performance of certain greedy schedulers, separately from the rest of the system. Here, we show that LPF can be used to characterize the performance of cross-layer schemes, spanning multiple layers. Some interesting future directions include analyzing the performance of GECS under fading channels, and under more general interference models.

## APPENDIX A

### PROOF OF THEOREM 3

Due to space considerations, we provide a sketch of the proof here and leave the detailed proof to the technical report [19]. We first provide the following Lemma that shows the existence of a stationary policy that keeps the queues stable.

**Lemma 1.** For any arrival rate  $\vec{\lambda} \in \sigma^* \Lambda_{P_{av}}$ , there is a stationary policy  $\Phi^S$  such that  $\lambda_l < \sigma^* \mu_l(\Phi^S), \forall l \in \mathcal{E}$ , and  $P_l^{av} \geq P_l(\Phi^S), \forall l \in \mathcal{E}$ .

We prove Theorem 1 by identifying a Lyapunov function and showing that its drift is negative for the fluid limit model of the system. Let  $\mathcal{A}_l(t)$  be the process representing the cumulative arrivals upto time  $t$ , and  $\mathcal{S}_l(t)$  be the cumulative

service process for queue  $Q_l$ . Similarly let  $\mathcal{W}_l(t)$  and  $\mathcal{Y}(t)$  be the cumulative arrival and service process for the virtual queue  $U_l(t)$ . We define  $G_{li}(t)$  as the number of time slots upto time  $t$  that link  $l$  was served with power level  $P_l^i$ . Let  $G_l(t)$  be the number of time slots upto time  $t$  that link  $l$  was selected by GECS in step 1 of the description of the GECS policy given in section III. Consider the sequence of scaled processes  $\{\frac{1}{n}\mathcal{A}_l(nt), \frac{1}{n}\mathcal{S}_l(nt), \frac{1}{n}Q_l(nt), \frac{1}{n}\mathcal{W}_l(nt), \frac{1}{n}\mathcal{Y}(nt), \frac{1}{n}U_l(nt), \frac{1}{n}G_l(nt), \frac{1}{n}G_{li}(nt)\}$ . Then, there exists a subsequence  $x_n$  such that  $\frac{1}{x_n}\mathcal{A}_l(x_nt) \rightarrow \lambda_l t$ ,  $\frac{1}{x_n}\mathcal{S}_l(x_nt) \rightarrow s_l(t)$ ,  $\frac{1}{x_n}Q_l(x_nt) \rightarrow q_l(t)$ ,  $\frac{1}{x_n}\mathcal{W}_l(x_nt) \rightarrow w_l(t)$ ,  $\frac{1}{x_n}\mathcal{Y}(x_nt) \rightarrow P_l^{av}t$ ,  $\frac{1}{x_n}U_l(x_nt) \rightarrow u_l(t)$ ,  $\frac{1}{x_n}G_l(x_nt) \rightarrow g_l(t)$ , and  $\frac{1}{x_n}G_{li}(x_nt) \rightarrow g_{li}(t)$  w.p. 1. The fluid limits are absolutely continuous, and satisfy:

$$\frac{d}{dt}q_l(t) = \begin{cases} [\lambda_l - \mu_l(t)]^+, & \text{if } q_l(t) > 0; \\ 0 & \text{o.w} \end{cases} \quad (3)$$

$$\frac{d}{dt}u_l(t) = \begin{cases} [\hat{w}_l(t) - P_l^{av}]^+, & \text{if } u_l(t) > 0, \\ 0 & \text{o.w} . \end{cases} \quad (4)$$

$$s_l(t_2) - s_l(t_1) = \sum_i C_l^i [g_{li}(t_2) - g_{li}(t_1)], \text{ if } q(t) > 0; \quad (5)$$

$$\int_0^t \mu_l(t) = \sum_i C_l^i g_{li}(t); \text{ if } q(t) > 0; \quad (6)$$

$$\int_0^t \hat{w}_l(t) = \sum_i P_l^i g_{li}(t); \quad (7)$$

$$g_l(t_2) - g_l(t_1) = \sum_i g_{li}(t_2) - g_{li}(t_1); \quad (8)$$

$$\dot{g}_l(t) = \sum_i \dot{g}_{li}(t); \quad (9)$$

Here,  $\mu_l(t) = \frac{d}{dt}s_l(t)$  is the service rate into queue  $q_l$ , whereas  $\hat{w}_l(t) = \frac{d}{dt}w_l(t)$  is the arrival rate into queue  $u_l$ . Let  $\mathcal{M}_{L(t)}$  be the set of link activation vectors on the set  $L(t)$ , as given in Definition 2. We define the projection of a vector  $\vec{a}$  on a set of edges  $L$ , denoted by  $\vec{a}|_L$  as  $\mathbf{P}\vec{a}$ , where  $\mathbf{P}$  is a  $|L| \times |\vec{a}|_1$  matrix such that  $P_{ij} = 1$ , if  $i = j$  and 0 otherwise.

**Lemma 2.** Consider the vector  $\vec{g}(t) = \langle \dot{g}_l(t) \rangle$ , i.e., its  $l^{\text{th}}$  element being  $\dot{g}_l(t)$ . Then,  $\vec{g}(t)$ , when projected on the set  $L(t)$  is a convex combination of link activation vectors in  $\mathcal{M}_{L(t)}$ .

**Lemma 3.** Under the GECS policy, for each link  $l$ , if there exist power levels  $i$  and  $j$  such that  $q_l(t)C_l^i - u_l(t)P_l^i > q_l(t)C_l^j - u_l(t)P_l^j$ , then  $\dot{g}_{lj}(t) = 0$ .

We now consider the Lyapunov function,  $V(\vec{q}(t), \vec{u}(t)) = \max_{l \in \mathcal{E}} \frac{1}{2}(q_l^2(t) + u_l^2(t))$ . Then, for any  $l \in L(t)$ ,

$$\frac{d}{dt}(q_l^2(t) + u_l^2(t)) \quad (10)$$

$$= q_l(t)(\lambda_l - \mu_l(t)) + u_l(t)(\hat{w}_l(t) - P_l^{av}) \quad (11)$$

$$= q_l(t)(\lambda_l - \sigma^* \mu^S) - u_l(t)(P_l^{av} - \hat{w}_l^S) + q_l(t)\sigma^* \mu_l^S - u_l(t)\hat{w}_l^S - [q_l(t)\mu_l(t) - u_l(t)\hat{w}_l(t)] \quad (12)$$

$$\leq q_l(t)(\lambda_l - \sigma^* \mu^S) - u_l(t)(P_l^{av} - \hat{w}_l^S) + \sigma^* q_l(t)\mu_l^S - \sigma^* u_l(t)\hat{w}_l^S - [q_l(t)\mu_l(t) - u_l(t)\hat{w}_l(t)] \quad (13)$$

By Lemma 1, we have  $\lambda_l - \sigma^* \mu^S < 0$  and  $P_l^{av} - \hat{w}_l^S \leq 0$ . The

following lemma is the final step in establishing the negative drift of the Lyapunov function.

**Lemma 4.** There exists a link  $\tilde{l} \in L(t)$  such that  $q_{\tilde{l}}(t)\mu_{\tilde{l}}(t) - u_{\tilde{l}}(t)\hat{w}_{\tilde{l}}(t) \geq \sigma^*(q_{\tilde{l}}(t)\mu_{\tilde{l}}^S - u_{\tilde{l}}(t)\hat{w}_{\tilde{l}}^S)$ .

The proof of Lemma 4 is given in [19]. Lemma 4 shows that there exists a link  $l \in L(t)$  such that  $\frac{d}{dt}q_l^2(t) + u_l^2(t) < 0$ . Considering the Lyapunov function  $V(\vec{q}(t), \vec{u}(t))$ , we have  $\frac{d}{dt}V(\vec{q}(t), \vec{u}(t)) \leq \max_{l \in L(t)} \frac{1}{2} \frac{d}{dt}(q_l^2(t) + u_l^2(t)) < 0$ . This establishes the negative drift of the Lyapunov function and hence the fluid limit model of the system is stable. Then, the corresponding queuing system is also stable as a consequence of Theorem 4 [18].

## REFERENCES

- [1] R. L. Cruz and A. Santhanam, "Optimal Routing, Link Scheduling, and Power Control in Multi-hop Wireless Networks", IEEE Infocom 2003.
- [2] R. Bhatia and M. S. Kodialam, "On Power Efficient Communication over Multi-hop Wireless Networks: Joint Routing, Scheduling and Power Control", IEEE Infocom 2004.
- [3] M. J. Neely, "Energy optimal control for time-varying wireless networks", IEEE Transactions on Information Theory, July 2006.
- [4] A. L. Stolyar. 2005. Maximizing Queuing Network Utility Subject to Stability: Greedy Primal-Dual Algorithm. Queuing Syst. Theory Appl. 50, 4 (August 2005), 401-457.
- [5] C. Li and M. J. Neely. 2010. Energy-Optimal Scheduling with Dynamic Channel Acquisition in Wireless Downlinks. IEEE Transactions on Mobile Computing 9, 4 (April 2010)
- [6] J. W. Lee, R. Majumdar, N. B. Shroff, "Downlink power allocation for multi-class wireless systems", IEEE/ACM Trans. Netw. 13(4): 854-867 (2005)
- [7] L. Lin, X. Lin, and N. B. Shroff, "Low-complexity and distributed energy minimization in multihop wireless networks," IEEE/ACM Trans. Netw. 18, 2 April 2010,
- [8] A. Dimakis and J. Walrand, "Sufficient conditions for stability of Longest-Queue-First scheduling: Second order properties using fluid limits," Advances in Applied probability, vol. 38, no. 2, pp.505-521, 2006.
- [9] G. Sharma, N. B. Shroff, and R. R. Mazumdar, "On the Complexity of Scheduling in Wireless Networks," ACM MobiCom, September 2006.
- [10] C. Joo, X. Lin and Ness. B. Shroff, "Understanding the capacity region of the Greedy maximal scheduling algorithm in multihop wireless networks", IEEE/ACM Trans. Netw. 17, August 2009.
- [11] C. Joo, X. Lin and Ness. B. Shroff, "Greedy Maximal Matching: Performance Limits for Arbitrary Network Graphs Under the Node-exclusive Interference Model" IEEE Transactions on Automatic Control, vol. 54, no. 12, pp. 2734-2744, Dec. 2009.
- [12] G. Zussman, A. Brzezinski, and E. Modiano, "Multihop Local Pooling for Distributed Throughput Maximization in Wireless Networks," in IEEE INFOCOM, Apr. 2008.
- [13] M. Leconte, J. Ni, and R. Srikant, "Improved bounds on the throughput efficiency of greedy maximal scheduling in wireless networks," ACM MobiHoc 2009.
- [14] Bo Li, Chem Boyaci and Ye Xia, "A Refined Performance Characterization of Longest-Queue-First Policy in Wireless Networks," Proceedings of ACM MobiHoc, 2009.
- [15] B. Birand, M. Chudnovsky, B. Ries, P. Seymour, G. Zussman, and Y. Zwols, "Analyzing the performance of greedy maximal scheduling via local pooling and graph theory." In Proc. INFOCOM'10.
- [16] L. Tassiulas, and A. Ephremides, "Stability properties of constrained queuing systems and scheduling policies for maximum throughput in multi-hop radio networks," IEEE Transactions on Automatic Control, pages 1936-1948, December 1992.
- [17] J. G. Dai, "On positive Harris Recurrence of Multiclass Queuing Networks: A Unified approach via Fluid Limit Models", Annals of Applied Probability, vol. 5, no. 1, pp. 49-77, 1995.
- [18] M. Andrews, K. Kumaran, K. Ramanan, A. Stolyar, R. Vijayakumar, P. Whiting, "Scheduling in a Queuing System with Asynchronously Varying Service Rates", Probability in the Engineering and Informational Sciences, Volume 18, Issue 2, April 2004, pages 191 - 217.
- [19] A. Sridharan, C. Joo, and C. E. Koksal, "Energy Efficient Greedy Link Scheduling and Power Control in wireless networks", Technical Report, url=<http://www.ece.osu.edu/~sridhara/greedyschedulingreport.pdf>