A Novel Coupled Queueing Model to Control Traffic via QoS-Aware Collision Pricing in Cognitive Radio Networks

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***Abstract*—We consider a cognitive radio network, where primary users have priority over the spectrum resources, and secondary users can exploit the unused resources through channel sensing. Due to sensing inaccuracy, the secondary traffic may obstruct the primary traffic. A penalty for collision has been used to protect the primary traffic, which is often designed to provide a fixed per-collision compensation or to restrict the collision rate at an acceptable level. In this work, we develop a framework that can protect the primary traffic taking into account the *Quality of Service of the primary traffic*. In particular, we pay attention to the delay performance, which is determined not only by the collision rate but also by the amount of traffic in both networks. We design a novel model with coupled queues, and successfully incorporate dynamic interactions between the two systems through the standard optimization problem. We also consider the practical requirement of *no direct sharing of the system information between the two networks*, and develop a close-to-optimal solution of per-collision price and channel sensing under mild assumptions. We evaluate its performance through simulations.**

I. Introduction

The significant underutilization of licensed frequency spectrum has resulted in numerous efforts to opportunistically exploit spatial and temporal spectrum gaps that are unused by primary (licensed) users [1]. In cognitive radio (CR) systems, using dynamic spectrum access (DSA), secondary (unlicensed) users are allowed access to the licensed spectrum band without interfering with the primary (licensed) users in order to improve spectrum efficiency.

Spectrum sensing is the key technology that enables a secondary user (SU) to access the unused licensed spectrum and reduces interference to the primary user (PU). In this work, we consider *in-band* spectrum sensing, where the SUs detect the PU signals in the channel that they are currently using [2]. Several sensing methods have been developed to detect the in-band PU signal: energy detection, feature detection, and matched filter detection. Energy detection has been widely used in signal detection. It detects a signal based on the total received energy level and does not need a priori knowledge of the PU signals [3]. Feature detection captures a specific signature of the PU signal such as pilot, preamble, or cyclostationarity. Matched filter detection can be performed by correlating the observed signal with the transmitted signal when the transmitted PU signal is exactly known.

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Although energy detection is easy to implement at low cost in either hardware or software [4], it could suffer from performance degradation under fading and unknown noise profiles. Matched filter detection and feature detection provide more accurate sensing, but they require additional information about the PU signal, which may not be available in practice. In this work we assume energy detection due to its popularity and low cost. Also it has been shown in [2] that energy detection has the attractive property of being able to meet the sensing requirements under noise uncertainty and interference.

In the presence of spectrum uncertainty, sensing accuracy significantly impacts the performance of both the primary and the secondary networks. A *miss detection* can incur significant interference to the PU traffic causing a collision, and a *false alarm* can result in a loss of spectrum opportunity. Note that the secondary network can control the sensing accuracy by adjusting the sensing time: the longer the SUs sense the channel, the more accurate the sensing results are. However, since longer sensing time immediately implies less time for data transmission, there is a trade-off between the sensing accuracy and the throughput of the secondary network. Further, the lack of sensing accuracy may cause the SUs to not detect the presence of a primary, resulting in degradation of the Quality of Service (QoS) of the primary traffic. In order to guarantee a certain level of service quality in the primary network, it also needs a means to control the secondary traffic or the sensing accuracy. To this end, we adopt a per-collision penalty charging mechanism [5], such that a collision penalty is used to coerce the secondary network in reducing its traffic amount and/or in improving its sensing accuracy.

There have been many works that study the system behavior under a collision penalty mechanism. However, most of them focus on the behavior of one network under the assumption of statistically static behavior of the other network. In this work, we develop new model that considers the dynamics of both the networks together, and takes into account the QoS of the primary network and the demand response of the secondary network. Given their priority to resources, we assume that the primary traffic and the secondary traffic have different traffic characteristics, which result in different utility functions. The performance of the two networks are connected through the shared spectrum resources and per-collision payment. We assume that there is a central management entity for each network, denoted by the primary provider (PP) and the secondary provider (SP), respectively. The PP sets the per-collision penalty price based on the availability of resources in the primary network and the amount of performance degradation of the PU traffic. The SP controls the transmission rate of SUs and adjusts the sensing time. We assume no direct communications between the two networks, except through the collision price and the long-term transmission rate of the primary traffic. Since the two networks are coupled through the spectrum resources and the sensing accuracy, it is not straightforward for them to find an equilibrium point without explicit system information from each other. Nonetheless, through novel coupled queueing model, we are able to develop a close-to-optimal solution that does not necessitate such information exchange.

In the literature, many resource allocation schemes have been proposed for cognitive radio networks. The basic trade-off between sensing and throughput has been studied in [6], where an optimal sensing time has been obtained to maximize the throughput of the secondary network. Cooperative sensing has been used to improve the sensing accuracy [7], [8]. An optimal Bayesian decision rule has been developed [9], and distributed cooperation of multiple SUs without a central entity has been developed in [10]. For efficient spectrum use over a relatively long-term period, auction-based schemes have been used in [11]. In an auction, the PP sells the spectrum opportunity, each SU bids for the resources, and the auction mechanism decides which SU can use the auction and charge an appropriate price such that the SUs reveal the true value of their resources. Most auctions consider spectrum pricing under user demand response with multiple providers and multiple users [12], [13]. Dynamic collision pricing under user demand response has been studied in [5] through the standard dual problem of the SU utility maximization when the miss detection probability is constrained by a certain level.

Our work is different from the previous studies in the following sense: i) We explicitly consider the performance degradation of the primary network to limit the interference from the secondary network, instead of imposing artificial constraints on the sensing accuracy. ii) We do not assume perfect knowledge of the spectrum opportunity and consider dynamic pricing over the interference, which will be determined by demand response of the SUs and the impact on the QoS of the primary network. iii) We assume that there is no direct exchange of system information between the primary and the secondary networks. This is an important practical feature, since, unless the two networks are operated by a single service provider, it is unlikely that they will share internal system information, such as link utilization and sensing accuracy.

The main contribution of the paper can be summarized as follows.

- We consider traffic characteristics of the primary and the secondary networks in accordance with their access priority. By introducing a system model with two coupled queues, we formulate a utility maximization problem that accounts for the delay performance of the primary network and the demand response of the secondary network under per-collision penalty charge mechanism.
- We investigate the dynamic behavior of the two networks, and develop a close-to-optimal solution to collision pricing and channel sensing under mild assumptions on miss detection probability and the concave utility of the SUs. We show that the proposed solution can be implemented with no explicit exchange of the detailed system information from the other network.
- Through simulations, we verify the operations of the proposed schemes and evaluate their performance.

The paper is organized as follow. The system model is provided in Section II. The introduction of the coupled queueing model and associated problem formulation has been described in Section III. A close-to-optimal solution for collision pricing and channel sensing has been developed in Section IV. Through simulations, we verify our approximation and evaluate the performance of the proposed scheme in Section V. Finally, we conclude our paper in Section VI.

II. System Model

We consider a cognitive radio network that consists of a primary service provider (PP) and a secondary service provider (SP). There is one primary user (PU), who has priority to access the licensed channel, and a set S of secondary users (SU). Time is slotted and one frequency channel is shared by all the users. We consider the *interweave* paradigm of spectrum sharing [14], where an SU can access the licensed channel only when the channel is not used by the PU. During a time slot, each SU senses the channel and reports the result to the SP. The SP fuses the information and makes a decision on the channel state. If the SP detects an idle channel, it schedules the time slot to the SU with the longest queue. Such a scheduling policy has been known to be throughput optimal in a single-hop network [15], [16]. An example scenario is LTE-based cognitive radio networks [17], where base stations of the primary networks and the secondary networks coexist, and the quality of service of the PU is more important than that of the SUs. In particular, we assume that the primary traffic is *inelastic* and *delay-sensitive* (e.g., Voice over LTE), and the secondary traffic is *elastic* and *throughput-sensitive* (e.g., file transfer and adaptive-rate multimedia transmission). The reason is that the primary traffic has unfettered access to the resources, while the secondary user may have to interrupt itself while the primary user is transmitting. Each SU joins the secondary network by subscribing the service of the SP and thus is assumed to be under the control of the SP.

Let $0 \leq \tau \leq 1$ denote the fraction of the time slot used for sensing and reporting. We assume that the sensing accuracy of the SP depends on τ. To elaborate, let $p_m = f(\tau)$ and $p_f = g(\tau)$ denote the probability of miss detection and the probability of false alarm, respectively. It has been shown in [2], [6] that the probabilities under energy detection schemes can be modeled through the Q function:

$$1 - f(\tau) = Q\left(\frac{1}{\sqrt{2 \cdot \text{SNR} + 1}}(Q^{-1}(\theta_f) - \sqrt{\tau f_s} \cdot \text{SNR})\right),$$
$$g(\tau) = Q\left(\sqrt{2 \cdot \text{SNR} + 1} \cdot Q^{-1}(1 - \theta_d) + \sqrt{\tau f_s} \cdot \text{SNR}\right),$$

where (θ_d, θ_f) is the target probabilities of miss detection and false alarm, SNR is the received signal-to-noise ratio, f_s is the sampling rate, and $Q(x)$ is the complementary distribution function given as $Q(x) := \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy$. As the sensing time τ increases, the probabilities of miss detection and false alarm decrease very quickly. In this work, we approximate[1] the rapid decrease of the probabilities by exponential decreasing functions, i.e.,

$$\begin{aligned} f(\tau) &= (p_m^{\max} - p_m^{\min}) \cdot e^{-c_m \tau} + p_m^{\min}, \\ g(\tau) &= (p_f^{\max} - p_f^{\min}) \cdot e^{-c_f \tau} + p_f^{\min}, \end{aligned} \tag{1}$$

where $p_m^{\min}$ and $p_m^{\max}$ are the upper and the lower bounds on p_m, $p_f^{\min}$ and $p_f^{\max}$ are the bounds on p_f, and $c_m, c_f > 0$ are a constant. We note that both the probabilities may not decrease to zero even if the whole time slot is used for sensing.

When an SP generates a false alarm, the time slot is wasted. When it misses detection, both transmissions of PU and SU collide and fail. In most previous studies, the SP (or the SUs) controls the secondary traffic to meet a certain requirement for the primary traffic (e.g., the collision rate should be less than a value [5], [19]), or to maximize its revenue after paying per-collision penalty to the PP [20]. In either case, we can formulate the objective function of the primary networks through a utility that includes the service fee of the primary traffic and the per-collision payment caused by the sensing errors of the SU. We assume that packets of unit size arrive at the PU following a Bernoulli process with rate λ_p, and during a time slot, one packet can be transmitted in the primary network.

$$\tilde{U}_p := v_p \cdot \lambda_p + \gamma \cdot c, \tag{2}$$

where v_p is unit gain of serving PU traffic, γ is the per-collision price paid by the SP, and c is the collision rate.

As mentioned in the Introduction, unlike the primary traffic that is supposed to be transmitted immediately after generation, we expect the secondary traffic is elastic, since it can be transmitted only when there is no primary traffic, which is hard to predict. We formulate the objective function of the secondary networks taking into account the diminishing returns of the user satisfaction for the throughput performance. Suppose that each SU i generates the traffic following an independent Bernoulli process with rate $\lambda_{s,i}$. Once a packet is transmitted, it leaves the network.

$$\tilde{U}_s := \sum_{i \in S} v_{s,i} \cdot \lambda_{s,i}^{\alpha} - \gamma \cdot c, \tag{3}$$

where $v_{s,i}$ denotes the utility weight of SU i and $0 < \alpha < 1$ denotes the concavity of individual utilities. The last term denotes the payment of the SP due to collision.

Although the model described by (2) and (3) is of interest, it does not precisely take into consideration the quality of the primary traffic in terms of delay, and thus its solution cannot fully utilize the network resources. The model has missed two key factors: i) *the quality loss of the primary traffic depends on the traffic amount in the primary and the secondary networks*, and ii) *the collision rate c is a function of the amount of traffics λ_p and $\{\lambda_{s,i}\}$*. For example, given a fixed rate of the primary traffic, as the secondary traffic increases, the waiting time of a primary packet will increase due to retransmissions (caused by a collision) and queueing, and thus, the per-collision penalty should change accordingly. Similarly, given a fixed rate of the secondary traffic, the collision rate will depend on the amount of the primary traffic, and thus the primary user's perception of the delay performance will be different according to the primary traffic load. Hence, it is necessary to develop a framework such that the collision price that is paid to the PP by the SP is determined according to the performance degradation of the primary traffic.

Another interesting point is that the SP can also control the collision rate by improving sensing accuracy through the sensing period τ. Given a transmission opportunity, the chosen SU can adjust its sensing time and improve the sensing accuracy at the expense of the amount of transmitted data, which will decrease the collision rate and may impact on the price decision. We need a new model that can take into account the impact of sensing time τ on the amount of the secondary traffic and the collision rate.

In the CR networks, it is often assumed that the primary network and the secondary network do not directly communicate with each other. Although they may share certain long-term information such as traffic statistics, they may not communicate at the time scale of microseconds or milliseconds. Hence, it is important to design a solution such that the PP and the SP can operate without the information of each other as well as they achieve good performance. In this work, we aim to develop an optimal solution that has low complexity and low control overhead, and can operate without direct communications between the PP and the SP. In the following, by introducing a novel queueing model, we formulate the problem that takes into account the QoS of the primary network and the sensing time of the secondary network.

III. Formulation with Two Dependent Queues

We start with modeling relatively long-term behavior of the CR networks. We consider two coupled queues. One is for the primary network with service rate μ_p, and the other for the secondary network with service rate μ_s. From basic queueing theory, we have $\rho_p := \frac{\lambda_p}{\mu_p}$ and $\rho_s := \frac{\lambda_s}{\mu_s}$ that are the busy

[1] Since the Q function can be approximated by $O(e^{-x^2})$ [18], we use $Q(\sqrt{\tau}) = O(e^{-\tau})$.

time of each queue, respectively. Since the PU has a higher access priority to the channel, we can establish the following relationship between the two queueing systems,

$$\mu_p = 1 - \rho_s p_m, \quad \mu_s = (1 - \rho_p) \cdot (1 - p_f). \tag{4}$$

These service rates are the key to understanding the interplay between the primary and the secondary networks in our model. It says that the packets of the primary traffic are not served when there is a collision, i.e., when the SUs have data (ρ_s) and fail to detect the channel (p_m). Also, the packets of the secondary traffic are served only when the PU is idle $(1-\rho_p)$ and the idle channel is successfully detected $(1-p_f)$. Suppose that λ_p, p_m, p_f are fixed. From (4), we have

$$\rho_p = \frac{\lambda_p}{1 - \rho_s p_m}, \quad \rho_s = \frac{\lambda_s}{(1-\rho_p)\cdot(1-p_f)}. \tag{5}$$

Associating the two equations, we can obtain

$$\begin{aligned} \rho_p &= \frac{1}{2}\left(\Upsilon_p - \sqrt{\Upsilon_p^2 - 4\lambda_p}\right), \\ \rho_s &= \frac{1}{2p_m}\left(\Upsilon_s - \sqrt{\Upsilon_s^2 - \frac{4p_m}{1-p_f}\cdot\lambda_s}\right), \end{aligned} \tag{6}$$

where $\Upsilon_p = 1 + \lambda_p - \frac{p_m}{1-p_f}\cdot\lambda_s$ and $\Upsilon_s = 1 - \lambda_p + \frac{p_m}{1-p_f}\cdot\lambda_s$. Note that the models are for long-term behavior in steady state. We verify our model through simulations in Section V by comparing (6) with the time fractions that the two queues are non-empty in CR networks, i.e., the busy periods.

Remarks: i) The collision rate c can be obtained as $\rho_p\rho_s p_m$. ii) The service rate sum $\mu_p + \mu_s$ can be greater than 1. This is because $(1-\mu_p)$ captures the time fraction that the server of the primary network cannot serve its traffic, and $(1-\mu_s)$ presents the time fraction that the server of the secondary network cannot serve its traffic. When the two queues are empty, both the servers count the time as idle (i.e., the time that they can serve).

Based on our coupled queueing model, we redesign the objective function of each network to take into account the system dynamics. For the primary traffic, we modify (2) to

$$U_p := v_p \cdot \lambda_p - c_p \cdot \frac{1/\mu_p}{1-\rho_p} + \gamma \cdot \rho_p\rho_s p_m, \tag{7}$$

where c_p is unit cost of delay. The new second term accounts for average delay cost, which is approximated through the M/M/1 queueing system. It turns out that our approximation is an upper bound when the miss detection probability is reasonably small. We refer to Appendix A for details.

For the secondary network, we incorporate the overhead incurred by the sensing period τ. Given the transmission opportunity of unit time, if we use τ fraction of time for sensing, the actual transmission rate decreases by $(1-\tau)$ and the utility sum of the SUs becomes

$$U_s := \sum_{i\in S} v_{s,i}\cdot((1-\tau)\cdot\lambda_{s,i})^\alpha - \gamma\cdot\rho_p\rho_s p_m, \tag{8}$$

where the last term denotes the payment of the SP due to collision. This implies that in our model, SU i transmits traffic at rate $(1-\tau)\lambda_{s,i}$, and $\lambda_{s,i}$ can be considered as the *traffic activity rate* that includes both data transmission and channel sensing. Thus, given traffic activity rate, τ determines actual data rate, as well as the probabilities of miss detection and false alarm.

In cognitive radio networks, unless both networks are operated by a single service provider, it is unlikely that the two networks share their detailed system information due to privacy reasons or sharing overhead. Hence, it is desirable that the PP and the SP can control their networks without direct coordination between them, and we assume that there is no explicit exchange of time-varying system information (i.e., μ_p, ρ_p, $\{\lambda_{s,i}\}$, μ_s, ρ_s, p_m, p_f, τ) between the primary and the secondary networks.

The two networks will try to maximize their own objective functions (7) and (8), respectively. We assume that both the PP and the SP have common knowledge on the form of the utility functions with some fixed parameters of v_p, c_p, α. The PP is assumed to know its own system parameters of $\lambda_p, \mu_p, \rho_p, \gamma$, and collision rate $\rho_p\rho_s p_m$. Similarly, the SP knows its own system parameters of $\{\lambda_{s,i}\}, \mu_s, \rho_s, p_m, p_f, \tau$ and collision rate $\rho_p\rho_s p_m$. The SP also knows the collision price γ and the long-term arrival rate λ_p of the primary traffic.

The main control parameters are the per-collision price γ for the PP, and the activity rate $\{\lambda_{s,i}\}$ and the sensing time τ for the SP. The dependency between the two networks and the constraints on information sharing make it hard to design an optimal solution. In the following, we show that our novel coupled queueing model admits low-complexity approximation to the optimal solutions without requiring time-varying system information of each other network.

IV. Collision Pricing and Channel Sensing

Suppose that the sensing time τ is fixed. Given a collision price, we first find the amount of the secondary traffic activity rate $\{\lambda_{s,i}\}$, and estimate the optimal collision price γ. Finally, we show that the sensing time can be determined independently at the SP.

Given a fixed sensing time τ, we reformulate the optimization problem of the SP by replacing $v_{s,i}$ with $\hat{v}_{s,i} := v_{s,i}\cdot(1-\tau)^\alpha$:

$$U_s := \sum_{i\in S}\hat{v}_{s,i}\cdot\lambda_{s,i}^\alpha - \gamma\cdot\rho_p\rho_s p_m. \tag{9}$$

Once the PP determines γ, the SP will respond by adjusting $\{\lambda_{s,i}\}$ according to (9). If the PP increases γ, the SP will reduce the transmission rates of the SUs, and the PP will observe that both the delay cost and the payment from the SP decrease. We find the optimal γ^* of the PP by backward induction, expecting the response of the SP. Note that $\frac{d}{d\gamma}U_p(\gamma^*) = 0$: at γ^*, the change in the delay cost will equal the change in the collision payment. This lead to the following price control.

Collision price control: Under mild assumption of $p_m \ll 1-\alpha$, we have

$$\gamma^* \approx \frac{\kappa}{\kappa-1} \cdot \frac{c_p}{\lambda_p} \cdot \frac{1}{(1-\rho_p)^2}, \tag{10}$$

where

$$\kappa := \frac{1}{1-\alpha} \cdot \frac{1}{\mu_p} \cdot \left(1 - \frac{\rho_s \rho_p p_m}{(1-\rho_p)\cdot \mu_p}\right)^{-1}. \tag{11}$$

We refer to Appendix B for details. Note that the PP can estimate κ without any direct information from the secondary networks since $\rho_p \rho_s p_m$ is the collision rate and can be measured at the PP. Later, we verify our results through simulations.

We now obtain the sensing time τ that is set by the SP to maximize its utility sum. The larger the sensing time is, the smaller the miss detection and the false alarm probabilities are, and the smaller the per-slot amount of data transmission is. Let τ^* denote the optimal time fraction for the sensing. For increasing τ, the changes of the utility gain from sensing inaccuracy and the utility loss from sensing overhead are monotone. Hence, we find the optimal value τ^* from $\frac{dU_s}{d\tau}(\tau^*) = 0$. We consider that the miss detection probability p_m and the false alarm probability p_f are a differentiable monotonically decreasing function of τ as shown in (1).

We first note that the collision price (10) is not a direct function of τ, which means that γ^* will be determined without considering the sensing period. This is because the quality of the primary traffic is affected by the secondary traffic activity rate including data transmission and channel sensing, and implies that the problem of how much fraction of time is used for the channel sensing can be separated from the determination of the secondary traffic activity rate. Hence, in our model, given the primary traffic λ_p, the collision price γ and the secondary traffic activity rate $\{\lambda_{s,i}\}$ will be determined by (10) and (17). Then, parameters ρ_p and ρ_s will also follow.

In order to maximize (8), we find τ^* such that the marginal utility loss equals the marginal utility gain. The utility loss from the increased sensing overhead can be written as

$$\frac{d}{d\tau}\left(\sum_i v_{s,i} \cdot \lambda_{s,i}^{\alpha} \cdot (1-\tau)^{\alpha}\right) = \sum_i v_{s,i} \cdot \lambda_{s,i}^{\alpha} \cdot (1-\tau)^{\alpha} \cdot \frac{-\alpha}{1-\tau}.$$

The utility gain from the higher sensing accuracy can be written as

$$\frac{d}{d\tau}(\gamma \cdot \rho_p \rho_s p_m) = \gamma \cdot \rho_p \rho_s p_m \cdot \left(\frac{1}{p_m} \cdot \frac{dp_m}{d\tau}\right).$$

From $\frac{d}{d\tau}\left(\sum_i v_{s,i} \cdot \lambda_{s,i}^{\alpha} \cdot (1-\tau)^{\alpha}\right) = \frac{d}{d\tau}(\gamma \cdot \rho_s \rho_p p_m)$ at τ^*, and the fact that $\left(\sum_i v_{s,i} \cdot \lambda_{s,i}^{\alpha} \cdot (1-\tau)^{\alpha}\right)$ denotes average utility gain and $\gamma \cdot \rho_p \rho_s p_m$ denotes average payment cost, we obtain the optimal sensing time τ^* as

$$\tau^* = 1 + \alpha \cdot \frac{\text{SP's avg. utility gain}}{\text{SP's avg. payment cost}} \left(\frac{1}{p_m} \cdot \frac{dp_m}{d\tau}\right)^{-1}. \tag{12}$$

The important highlight here is that the SP can calculate (12) without any information from the PP. Note that the optimal sensing time (12) is obtained from fixed $\{\lambda_{s,i}\}$, ρ_p, ρ_s, γ^*. In practice, however, the change of τ will impact $\{\lambda_{s,i}\}$, ρ_s, ρ_p, γ^* through the sensing accuracy p_m and p_f. Nonetheless, we can apply (12) by restricting the per-slot change of τ to a sufficiently small value, such that the short-term changes of $\{\lambda_{s,i}\}$, ρ_s, ρ_p, γ^* are negligible. To elaborate, we employ the following additive control of the sensing time. Let $\hat{\tau}^*(t)$ denote our estimation using (12) at time t.

TABLE I
DEFAULT SIMULATION SETTINGS.

Primary networks	$\lambda_p = 0.5$, $v_p = 1$, $c_p = 2$.
Secondary networks	$v_{s,1} = v_{s,2} = 1$, $\alpha = 0.5$, $\delta = 0.001$, $p_m^{\max} = p_f^{\max} = 0.5$, $p_m^{\min} = p_f^{\min} = 0.05$, $c_m = c_f = 50$.

Sensing time control:

$$\tau(t+1) = \begin{cases} \tau(t) + \delta, & \text{if } \tau(t) \le \hat{\tau}^*(t), \\ \tau(t) - \delta, & \text{if } \tau(t) > \hat{\tau}^*(t), \end{cases} \tag{13}$$

where δ is a sufficiently small value. Note that (13) can be viewed as a time-scale separation such that $\tau(t)$ changes slowly compared to the other system parameters. In order to control the sensing time at the same speed as the other system parameters, the sensitivity of the parameters with respect to each other needs to be carefully addressed, which remains as an open problem.

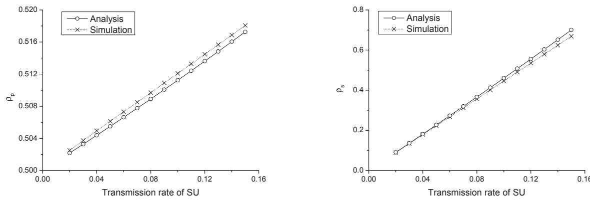
Our collision price control (10) and channel sensing control (13) are based on our novel coupled queueing model and successfully takes into account the delay performance of the primary traffic. In addition, they can be implemented without direct communications between the primary and the secondary networks. In the following section, we show through simulation that our solution provides good performance close to the optimal.

V. SIMULATION

We verify our coupled queueing model and evaluate the performance of our solution. We start with a simple network of one PU and two SUs, where, unless otherwise stated, the two SUs are set identically as in Table I.

Given λ_p, p_m, and p_f, we verify our model by showing that the busy periods ρ_p and ρ_s match with our analysis results (6). We measure the busy period of the two servers for different arrival rates of the secondary traffic, and observe that the measurement results are very close to the analysis results, as shown in Fig. 1

We now check the optimality of our collision pricing (10) and channel sensing control (13). We first show that our SU transmission rate (17) maximizes the utility of the SU. We fix the collision price $\gamma = 30$, the probabilities $p_m = p_f = 0.051$, and the sensing time $\tau = 0$. We measure the total utility, and compare the performance under fixed SU transmission rates. Fig. 2(a) shows the total utility of the SUs with different SU



(a) ρ_p of the primary network (b) ρ_s of the secondary network

Fig. 1. Comparison of busy periods (6) with simulation results. There are one PU with $\lambda_p = 0.5$ and two SUs with different arrival rates. The probabilities of miss detection and false alarm are fixed to $p_m = p_f = 0.051$.

transmission rates and that under our SU transmission rate (17), which is marked with a red dot. It shows that our setting is very close to the maximizer of the total utility. Fig. 2(b) shows the performance of the per-collision price control (10) at the primary network. The SU transmission rate is set by (17), and the probabilities and the sensing time are fixed to $p_m = p_f = 0.051$ and $\tau = 0$, respectively. We measure the utility of the primary network under fixed per-collision prices with the performance under our price control of (10). The results show that our control finds a close-to-optimal price, and slightly overestimates the optimal price due to estimation errors and our approximation (18). Finally, Fig. 2(c) illustrates the performance of the sensing time controller at the secondary network. In this experiment, the collision price is set by (10). The probabilities of miss detection and false alarm are now determined by the sensing time τ following (1), with the parameters shown in Table I. We compare the performance under fixed sensing times and under our control (12). The simulation results show that our control successfully maximizes the total utility of the SUs.

Next, we evaluate the system performance for different PU transmission rates λ_p. Since the primary traffic has priority over the secondary traffic, we expect that the SUs will suffer from low throughput as λ_p increases. Under our proposed solution, it is expected that the collision price will increase, the transmission rate of SUs will decrease, and the SU will spend more time for sensing for accurate sensing of idle channel states. The results shown in Fig. 3 demonstrate such behavior of the system, as λ_p increases.

When $\lambda_p = 0.5$, we have per-collision price $\gamma^* = 31.7644$ and sensing time $\tau^* = 0.121$. For comparison, we consider a static scheme with fixed per-collision price $\gamma = 31.7644$ and fixed sensing time $\tau = 0.121$. The static scheme can be considered as a solution to (2) and (3). Comparing the performance of our solution and the static scheme, under light primary loads, our solution provides lower prices and admits more secondary traffic, and vice versa. We also measure packet delays of the primary traffic. Fig. 4 shows that our solution achieves better delay performance under heavy traffic loads due to higher collision prices.

Fig. 5 illustrates the impact of α, which represents the concavity of user satisfaction for the secondary users. As α increases, the utility function gets closer to a linear function, and thus the SUs are more likely to increase their transmission rate. However, beyond a certain point, the rate starts to decrease[2]. For the per-collision price, it is inversely proportional to α from (11) since $\frac{\kappa}{\kappa-1} \approx \frac{1}{\alpha}$ as discussed in Appendix C. The total utility of the SUs also decreases due to larger α (because $\lambda_{s,i} < 1$). The sensing time does not change much; 0.121 for all $\alpha \geq 0.3$.

In the next experiment, we consider four SUs with different weights, i.e., $v_{s,1} = 1$, $v_{s,2} = 0.7$, $v_{s,3} = 0.4$, and $v_{s,4} = 0.2$, and measure their transmission rates. The other settings follow Table I. Fig. 6 shows how α impacts the overall system performance and the fairness between the SUs. When α is too small, the SUs share the resource in a fair manner but suffer from low performance. When α is too large, one SU dominates the others. The result demonstrate that α near 0.5 provides good balance of the two performance metrics.

VI. Conclusion

In this work, we develop a novel framework that accounts for both the Quality of Service of the primary network and the demand response of the secondary network. We develop novel coupled queueing model and successfully incorporate the dynamics of the two networks in the standard formulation of utility maximization.

We investigate the behavior of the two networks under spectrum uncertainty and sensing inaccuracy. The PP adjusts per-collision price to protect the primary traffic, and the SP optimizes sensing time to balance the transmission gain and the collision cost. Under mild assumptions of low miss detection probability and heavily concave utility of the SUs, we develop separate controllers that achieve close-to-optimal performance and do not require system information of the other network, which is of importance in practice. Through simulations, we verify the operations of our control design and evaluate their performance.

There remain many interesting open questions. Under our proposed scheme, it is possible that $\rho_p + \rho_s \geq 1$ and the queue lengths of the SUs may grow. Since excessively long delay is not desirable even for elastic traffic, we need additional constraints or control variables to restrict the queue lengths and to provide stability. Also, when there are multiple primary and secondary networks, it is would be interesting to see how they compete to attract the users and to optimize their performance. A game theoretic approach would be helpful in this direction.

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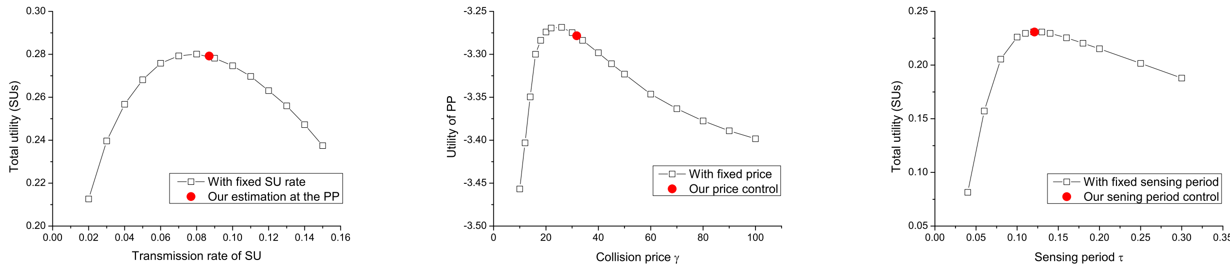
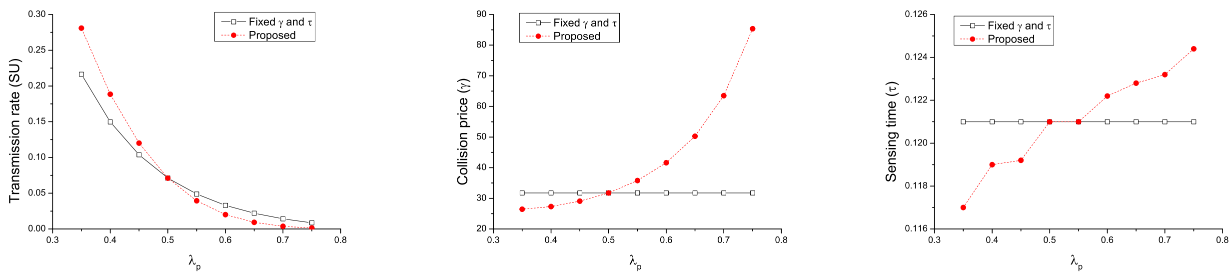
[2]From (17), the transmission rate is close to $(\alpha\beta)^{1/(1-\alpha)}$ when α is close to 1. If $\beta < 1$, the function starts to decrease as α gets closer to 1, which explains why the transmission rate decreases near $\alpha = 1$. We note that as $\alpha \to 1$, our assumption of $p_m \ll 1 - \alpha$ no longer holds and there will be larger approximation errors in (18).



(a) SU transmission rate $\lambda_{s,i}$ (17)
(b) Collision price γ^* (10)
(c) Channel sensing τ^* (12)

Fig. 2. Verification on the SU transmission rates $\{\lambda_{s,i}\}$, the per-collision price γ^* and the sensing period τ^*.



(a) SU transmission rate
(b) Collision price
(c) Sensing time

Fig. 3. System parameters of the proposed scheme with different PU transmission rates λ_p. As λ_p increases, the resources for the secondary network become scarcer and the collision price increases, which reduces the transmission rate of the SUs. Also, due to higher collision price, the sensing time expands for more accurate sensing. In comparison with our scheme, the static scheme with fixed γ and τ achieves lower secondary traffic under light primary traffic load and more secondary traffic under heavy primary traffic load.

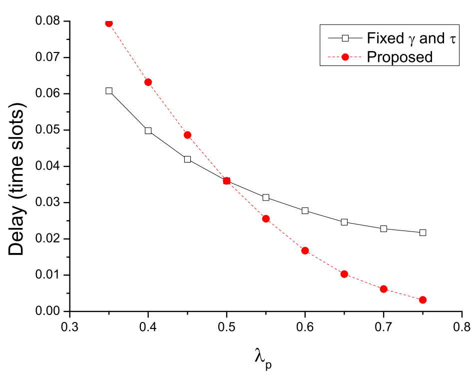


Fig. 4. Average delay of the primary traffic.

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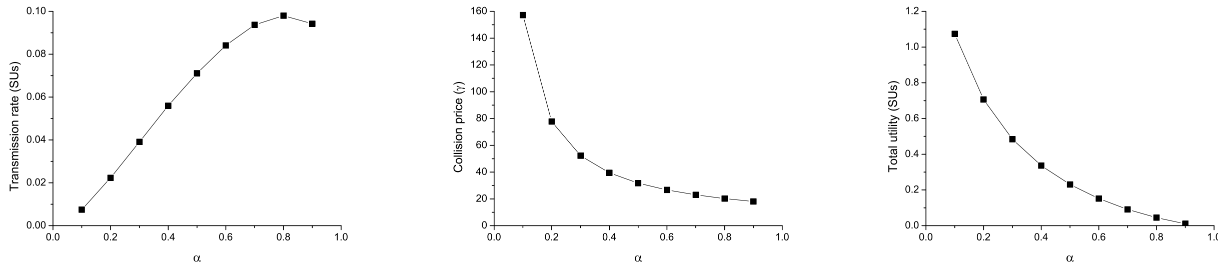

(a) SU transmission rate (b) Collision price (c) Total utility of SUs

Fig. 5. Sensitivity on α. The transmission rate of SU linearly increases when α is small, and the collision price is inversely proportional to α. The utility of the SU decreases since $\lambda_{s,i} \leq 1$.

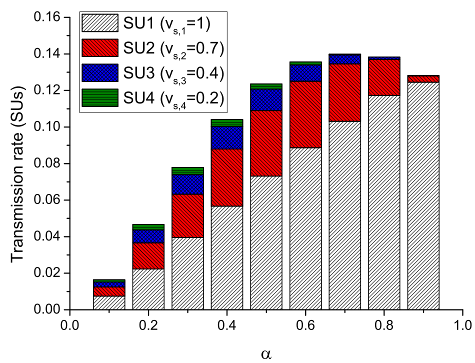

Fig. 6. Simulation with four SUs of different weights $v_{s,i}$. It clarifies the trade-off of α between the throughput and the fairness in the secondary network. Settings around $\alpha = 0.5$ provide a good balance.

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Appendix

A. Delay bound using the M/M/1 queueing model

Let X denote the random variable for the service time of a packet in the primary network. At each time, since the primary network has the priority, the packet will be transmitted and will be successfully transmitted with probability no smaller than $1 - p_m$, which implies that

$$1 \leq E[X] \leq \frac{1}{1 - p_m}.$$

Further, the second moment of X is no greater than the second moment of the service time when the successful transmission probability exactly equals $1 - p_m$, i.e.,

$$E[X^2] \leq (1-p_m) + 2^2(1-p_m)p_m + 3^2(1-p_m)p_m^2 + \dots$$
$$= (1-p_m)\sum_{n=1}^{\infty} n^2 p_m^{n-1} = \frac{1+p_m}{(1-p_m)^2}.$$

From the Pollaczek-Khinchin formula, if $E[X^2] - E^2[X] = Var(X) \leq E^2[X]$, the queueing delay of M/G/1 is smaller than the queueing delay of M/M/1. Since $E[X] \geq 1$, the condition holds if $E[X^2] \leq 2$ or

$$\frac{1+p_m}{(1-p_m)^2} \leq 2,$$

which is true when $p_m \leq 0.219$.

Therefore, in our system, the M/M/1 queueing model becomes an upper bound on the delay when the miss detection probability is reasonably small, i.e., if $p_m \leq 0.219$. We note that IEEE 802.22 requires the miss detection probability smaller than 0.1 [2].

B. Collision price control

We first investigate the relationship between γ, ρ_p, and ρ_s in our coupled queueing model. Based on the response of the SP, we obtain the optimal price γ^* that maximizes the utility of PP.

The following intermediate results will be used later to calculate the gain and the loss of U_p. From (5), we have

$$\frac{d\rho_p}{d\rho_s} = \frac{\lambda_p p_m}{(1-\rho_s p_m)^2} = \rho_p \cdot \frac{p_m}{1-\rho_s p_m}. \tag{14}$$

Also, since $\mu_s = (1-\rho_p)\cdot(1-p_f)$, we have

$$\begin{aligned}\frac{d\rho_s}{d\lambda_s} &= \frac{1}{\mu_s} + \frac{\lambda_s}{\mu_s^2}\cdot\left(-\frac{d\mu_s}{d\lambda_s}\right)\\ &= \frac{1}{\mu_s} + \frac{\rho_s}{\mu_s}\cdot(1-p_f)\cdot\frac{d\rho_p}{d\rho_s}\cdot\frac{d\rho_s}{d\lambda_s}.\end{aligned} \tag{15}$$

Combining it with (14), we can obtain

$$\frac{d\rho_s}{d\lambda_s} = \frac{1}{\mu_s}\cdot\left(1 - \frac{\rho_s\rho_p p_m}{(1-\rho_p)\cdot\mu_p}\right)^{-1}. \tag{16}$$

Given γ, the transmission rate of SU i will be set to maximize (9), satisfying

$$\hat{v}_{s,i}\cdot\alpha\cdot\lambda_{s,i}^{\alpha-1} = \gamma\cdot p_m\cdot\frac{d}{d\lambda_{s,i}}(\rho_s\rho_p).$$

From (15), the transmission rate of SU i can be obtained as

$$\lambda_{s,i}^{1-\alpha} = \frac{\hat{v}_{s,i}}{\gamma \cdot p_m} \cdot \alpha \cdot \left(\frac{\rho_p}{1-\rho_s p_m} \cdot \frac{d\rho_s}{d\lambda_{s,i}} \right)^{-1} = \frac{\hat{v}_{s,i}}{\gamma \cdot p_m} \cdot \frac{\alpha}{1-\alpha} \cdot \frac{1}{\kappa} \cdot \frac{\mu_s}{\rho_p},$$

where $\frac{d\rho_s}{d\lambda_{s,i}} = \frac{d\rho_s}{d\lambda_s}$ for all $i \in S$. Note that from $0 < \alpha < 1$, $\mu_p \leq 1$, and $1 - \frac{\rho_s \rho_p p_m}{(1-\rho_p)\cdot\mu_p} \leq 1$, we have $\kappa > 1$. Hence, the transmission rate of SU i will be set as

$$\lambda_{s,i} = \left(\frac{\hat{v}_{s,i}}{\gamma \cdot p_m} \cdot \frac{\alpha}{1-\alpha} \cdot \frac{1}{\kappa} \cdot \frac{\mu_s}{\rho_p} \right)^{1/(1-\alpha)}. \tag{17}$$

From the measurements on μ_s and ρ_s, the SP can estimate μ_p and ρ_p from (4) and (5), and thus it can calculate κ and $\{\lambda_{s,i}\}$ with no system information from the PP. Also, at the optimal price γ^*, the following condition obtained by differentiating (17) should be satisfied.

$$\frac{d\lambda_{s,i}}{d\gamma} = -\frac{\lambda_{s,i}}{(1-\alpha)\gamma} \cdot \left(1 + \kappa \cdot \frac{\lambda_{s,i}/\mu_s}{1-\rho_p} \cdot p_m \right)^{-1} \approx -\frac{\lambda_{s,i}}{(1-\alpha)\cdot\gamma^*}, \quad \text{when } p_m \ll 1-\alpha. \tag{18}$$

For the approximation, we refer to Appendix C. In the sequel, we assume that $p_m \ll 1-\alpha$, which holds when the utility function is heavily concave.

Now, we are ready to estimate γ^* using $\frac{d}{d\gamma}U_p(\gamma^*) = 0$. Let dU_p^* denote the marginal gain of the collision payment when the collision price increases, i.e.,

$$dU_p^+ := \frac{d}{d\gamma}(\gamma \cdot \rho_p \rho_s p_m) = \rho_p \rho_s p_m + \gamma p_m \cdot \frac{\rho_p}{1-\rho_s p_m} \cdot \frac{d\rho_s}{d\lambda_s} \cdot \frac{d\lambda_s}{d\gamma}.$$

The last equality comes from $\frac{d\rho_p}{d\gamma} = \frac{d\rho_p}{d\rho_s} \cdot \frac{d\rho_s}{d\gamma}$ and (14). Using (16) and (18), we obtain

$$dU_p^+ \approx \rho_p \rho_s p_m \cdot \left(1 - \frac{1}{1-\alpha} \cdot \frac{1}{\mu_p} \cdot \left(1 - \frac{\rho_s \rho_p p_m}{(1-\rho_p)\cdot\mu_p} \right)^{-1} \right) = \rho_p \rho_s p_m \cdot (1-\kappa).$$

Since $\kappa > 1$, we have $dU_p^+ < 0$.

Similarly, let dU_p^- denote the change of the delay cost, which can be driven as

$$dU_p^- := \frac{d}{d\gamma}\left(c_p \cdot \frac{1/\mu_p}{1-\rho_p} \right) = \frac{c_p}{\lambda_p} \cdot \frac{d}{d\gamma}\left(\frac{1}{1-\rho_p} - 1 \right) = \frac{c_p}{\lambda_p} \cdot \frac{1}{(1-\rho_p)^2} \cdot \frac{d\rho_p}{d\rho_s} \cdot \frac{d\rho_s}{d\lambda_s} \cdot \frac{d\lambda_s}{d\gamma} \approx -\frac{c_p}{\lambda_p} \cdot \frac{1}{(1-\rho_p)^2} \cdot \rho_p \rho_s p_m \cdot \kappa \cdot \frac{1}{\gamma}.$$

From the fact that $-dU_p^- + dU_p^+ = 0$ at the optimal γ^*, we have

$$\gamma^* \approx \frac{\kappa}{\kappa - 1} \cdot \frac{c_p}{\lambda_p} \cdot \frac{1}{(1-\rho_p)^2}.$$

C. Approximation of $\frac{d\lambda_{s,i}}{d\gamma}$

From $\lambda_{s,i} = \left(\frac{\hat{v}_{s,i}}{\gamma\cdot p_m} \cdot \frac{\alpha}{1-\alpha} \cdot \frac{1}{\kappa} \cdot \frac{\mu_s}{\rho_p} \right)^{1/(1-\alpha)}$, we have

$$\frac{d\lambda_{s,i}}{d\gamma} = \frac{\lambda_{s,i}}{1-\alpha} \cdot \left(-\frac{1}{\gamma} + \frac{\rho_p}{\mu_s} \cdot \frac{d}{d\gamma}\left(\frac{\mu_s}{\rho_p} \right) \right).$$

Also, from (4), (14) and (16), the second term can be written as

$$\frac{\rho_p}{\mu_s} \cdot \frac{d}{d\gamma}\left(\frac{\mu_s}{\rho_p} \right) = -\frac{\rho_p}{\mu_s} \cdot \frac{1-p_f}{\rho_p^2} \cdot \frac{d\rho_p}{d\gamma} = -\frac{1}{\mu_s} \cdot \frac{1-p_f}{\rho_p} \cdot \frac{d\rho_p}{d\rho_s} \cdot \frac{d\rho_s}{d\lambda_{s,i}} \cdot \frac{d\lambda_{s,i}}{d\gamma} = -\frac{1}{(1-\rho_p)\rho_p} \cdot \frac{\rho_p p_m}{\mu_p} \cdot \frac{1}{\mu_s} \left(1 - \frac{\rho_p \rho_s p_m}{(1-\rho_p)\mu_p} \right)^{-1} \cdot \frac{d\lambda_{s,i}}{d\gamma}.$$

Combining the above two equations, we obtain

$$\frac{d\lambda_{s,i}}{d\gamma} = -\frac{\lambda_{s,i}}{(1-\alpha)\gamma} - \frac{\lambda_{s,i}}{1-\alpha} \cdot \frac{p_m}{(1-\rho_p)\mu_p} \cdot \frac{1}{\mu_s} \left(1 - \frac{\rho_p \rho_s p_m}{(1-\rho_p)\mu_p} \right)^{-1} \cdot \frac{d\lambda_{s,i}}{d\gamma}.$$

From (11), we have

$$\frac{d\lambda_{s,i}}{d\gamma} = -\frac{\lambda_{s,i}}{(1-\alpha)\gamma} \cdot \left(1 + \kappa \cdot \frac{\lambda_{s,i}/\mu_s}{1-\rho_p} \cdot p_m \right)^{-1}.$$

Note that $\frac{\lambda_{s,i}/\mu_s}{1-\rho_p} \leq \frac{\rho_s}{1-\rho_p} \leq 1$ when $\rho_p + \rho_s \leq 1$. Also, when p_m is small, the collision probability $\rho_p \rho_s p_m$ is small and $\mu_p = 1 - \rho_s p_m$ is close to 1, which implies $\kappa \approx \frac{1}{1-\alpha}$. Hence, when the miss probability satisfies $p_m \ll 1-\alpha$, we have

$$\frac{d\lambda_{s,i}}{d\gamma} \approx -\frac{\lambda_{s,i}}{(1-\alpha)\gamma}.$$