Joint Subcarrier Assignment and Power Allocation in Full-Duplex OFDMA Networks

Changwon Nam
School of EECS, INMC
Seoul National University
Email: cwnam@netlab.snu.ac.kr

Changhee Joo
School of Electrical and Computer Engineering
UNIST
Email: cjoo@unist.ac.kr

Saewoong Bahk
School of EECS, INMC
Seoul National University
Email: sbahk@snu.ac.kr

***Abstract*—Recent advances in communication technology have demonstrated the feasibility of full-duplex communication for a node to simultaneously transmit and receive on the same frequency band. Although the full-duplex technique has the potential to double the capacity of point-to-point wireless links, the resource allocation in multi-user environment becomes more complicated due to user-dependent channel conditions and per-user power constraints. In this paper, we formulate the joint problem of subcarrier assignment and power allocation in full-duplex OFDMA networks, and develop an iterative solution based on the necessary condition for optimality. We evaluate our solution through extensive simulations, which shows that it empirically achieves near-optimal performance and outperforms other resource allocation schemes designed for half-duplex networks.**

***Index Terms*—Full-duplex communications, OFDMA, wireless resource allocation.**

I. Introduction

Recent advances in signal processing have demonstrated the feasibility of wireless full-duplex communications, which enables a full-duplex node to transmit and receive simultaneously in the same frequency band by countervailing against the self-interference caused by its own transmission. The main difficulty in full-duplex communications is suppressing self-interference to a sufficiently low level. In the literature, extensive researches have been conducted for self-interference cancellation techniques, which can be categorized into antenna cancellation [1], analog cancellation [2], and digital cancellation [2]. The state-of-the-art work has demonstrated that self-interference can be suppressed to the noise floor level by the combination of multiple cancellation techniques [3].

Orthogonal Frequency Division Multiple Access (OFDMA) has been a key technology for multiple access in most wideband cellular networks [4], [5], which consist of a base station (BS) and multiple mobile nodes. By dividing the entire spectrum into multiple orthogonal subcarriers and distributing them over nodes, OFDMA can benefit from both multiuser and frequency diversities. Resource management in (half-duplex) downlink or uplink OFDMA systems has been extensively studied in the literature to maximize the sum-rate by assigning subcarriers and allocating transmission power under a limited power budget. It has been known that an optimal solution to the downlink resource allocation problem is a combination of the channel-based subcarrier assignment and the well-known water-filling power allocation [6]. On the other hand, the uplink problem is more challenging due to per-node power constraints, i.e., each node of uplink transmission has its own power budget. Most previous results achieve suboptimal performance [7], or solve a relaxed approximation problem [8].

The full-duplex capability enables the BS to transmit downlink traffic to nodes while receiving uplink traffic from them simultaneously. Since the uplink and downlink transmissions are no longer independent, previous solutions are unlikely to optimize the performance, thus necessitating new solutions that account for the characteristics of the simultaneous uplink and downlink transmissions. In this paper, we aim to maximize the full-duplex sum-rate by jointly optimizing subcarrier assignment and power allocation in the presence of full-duplex transmissions. We propose a joint solution to the subcarrier assignment and power allocation problem by establishing a necessary condition for the sum-rate optimality. Through extensive simulations, we show that our algorithm empirically achieves near-optimal performance and outperforms other resource allocation schemes.

The rest of this paper is organized as follows. The full-duplex sum-rate maximization problem is formally formulated in Section II, and a necessary condition for optimality is derived in Section III. Our proposed subcarrier assignment and power allocation algorithm is described in Section IV. We empirically evaluate our solution in comparison with other resource allocation schemes in Section V. Finally, we conclude our paper in Section VI.

II. System Model

We consider a single-cell full-duplex OFDMA network. There are one full-duplex base station (BS) and N full-duplex mobile nodes, and let $\mathcal{N} = \{1, 2, \ldots, N\}$ denote the set of nodes. The entire frequency band is partitioned into S subcarriers, and let $\mathcal{S} = \{1, 2, \ldots, S\}$ denote the set of subcarriers. Each subcarrier is perfectly orthogonal to each other without inter-subcarrier interference. We assume that self-interference can be successfully suppressed below the noise power level by various cancellation techniques [3]. Using the full-duplex capability, the BS exchanges downlink traffic and uplink traffic with each node simultaneously.

We assume that a subcarrier is exclusively assigned to a node for both uplink and downlink transmissions together. We denote a subcarrier assignment pattern by a binary vector $X := \{x_{n,s}\}_{n\in\mathcal{N},s\in\mathcal{S}}$, where element $x_{n,s}$'s are defined as

$$x_{n,s} = \begin{cases} 1, & \text{if subcarrier } s \text{ is assigned to node } n, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Let $u_{n,s}$ and $d_{n,s}$ denote the channel gains for subcarrier s (normalized with respect to the noise power) between the BS and node n for uplink and downlink, respectively, and let $G := \{u_{n,s}, d_{n,s}\}_{n\in\mathcal{N},s\in\mathcal{S}}$ denote the channel gain vector. Also, $p_{n,s}$ and $q_{n,s}$ denote the uplink and downlink powers allocated to subcarrier s for node n, respectively. Let $P := \{p_{n,s}\}_{n\in\mathcal{N},s\in\mathcal{S}}$ denote the uplink power allocation vector, and $Q := \{q_{n,s}\}_{n\in\mathcal{N},s\in\mathcal{S}}$ denote the downlink power allocation vector. The total transmission powers at the BS and node n are limited to P_{BS} and P_n, respectively.

The subcarrier sum-rate $R_{n,s}$ of node n in subcarrier s can be written as

$$\begin{aligned} &R_{n,s}(X,P,Q) \\ &= x_{n,s}\{\log(1+p_{n,s}u_{n,s}) + \log(1+q_{n,s}d_{n,s})\}. \end{aligned} \tag{2}$$

Let $R_n(X,P,Q)$ denote the sum-rate of node n over all subcarriers, i.e., $R_n(X,P,Q) = \sum_{s=1}^{S} R_{n,s}$ and let $R(X,P,Q)$ denote the total sum-rate, i.e., $R(X,P,Q) = \sum_{n=1}^{N} R_n$.

In this paper, our goal is to maximize the total sum-rate $R(X,P,Q)$ by jointly optimizing the subcarrier assignment X and the power allocation (P,Q) under the power constraints of the BS and each node. Then we formally formulate the full-duplex sum-rate maximization problem P as follows:

$$(\mathsf{P}) \ \text{maximize} \ R(X,P,Q) \tag{3}$$

$$\text{subject to} \ \sum_{n=1}^{N} x_{n,s} \le 1, \forall s \in \mathcal{S} \tag{4}$$

$$\sum_{s=1}^{S} p_{n,s} \le P_n, \forall n \in \mathcal{N} \tag{5}$$

$$\sum_{n=1}^{N}\sum_{s=1}^{S} q_{n,s} \le P_{BS}, \tag{6}$$

$$p_{n,s} \ge 0, \forall n \in \mathcal{N}, \forall s \in \mathcal{S} \tag{7}$$

$$q_{n,s} \ge 0, \forall n \in \mathcal{N}, \forall s \in \mathcal{S} \tag{8}$$

$$x_{n,s} \in \{0,1\}, \forall n \in \mathcal{N}, \forall s \in \mathcal{S}. \tag{9}$$

Note that each subcarrier is exclusively assigned to a single node according to (4) and (9).

III. Necessary Condition for Optimality

Due to the exclusive nature of subcarrier assignment, the original problem P is an integer optimization problem, which generally requires exponential complexity to be solved. Therefore, we relax the constraints and allow multiple nodes to share a subcarrier together. The binary constraints (9) are replaced with

$$x_{n,s} \ge 0, \forall n \in \mathcal{N}, \forall s \in \mathcal{S}. \tag{10}$$

From (4) and (10), we have $x_{n,s} \in [0,1]$.

The relaxed problem $\mathsf{P}^{'}$ obtained by replacing (9) with (10) is still not a convex problem because $R(X,P,Q)$ is not (jointly) concave in X and (P,Q). However, the optimal solution still satisfies the Karush-Kuhn-Tucker (KKT) condition [9], so we can obtain the following proposition.

Proposition 1. Let $X^* = \{x^*_{n,s}\}$, $P^* = \{p^*_{n,s}\}$, and $Q^* = \{q^*_{n,s}\}$ denote the optimal solution to problem $\mathsf{P}^{'}$. If $x^*_{n,s} > 0$, then we have

$$n = \underset{m\in\mathcal{N}}{\arg\max}\{\log(1+p_{m,s}u_{m,s}) + \log(1+q_{m,s}d_{m,s})\}. \tag{11}$$

Proof: We can define the Lagrangian function L as

$$\begin{aligned} &L(X,P,\lambda,\mu,\nu) \\ &:= \sum_{n=1}^{N}\sum_{s=1}^{S} x_{n,s}\{\log(1+p_{n,s}u_{n,s}) + \log(1+q_{n,s}d_{n,s})\} \\ &\quad + \sum_{s=1}^{S}\lambda_s\left(1-\sum_{n=1}^{N}x_{n,s}\right) + \sum_{n=1}^{N}\mu_n\left(P_n - \sum_{s=1}^{S}p_{n,s}\right) \\ &\quad + \nu\left(P_{BS} - \sum_{n=1}^{N}\sum_{s=1}^{S}q_{n,s}\right), \end{aligned}$$

where $\lambda = \{\lambda_s\}$, $\mu = \{\mu_n\}$, and ν are the dual variables (or the Lagrangian multipliers). Then X^* and (P^*, Q^*) should satisfy the following KKT conditions:

$$\begin{aligned} \frac{\partial L}{\partial x_{n,s}}\Big|_{X^*,P^*,Q^*} &= \log\left(1+p^*_{n,s}u_{n,s}\right) + \log\left(1+q^*_{n,s}d_{n,s}\right) \\ &\quad - \lambda_s \begin{cases} = 0, \text{if } x^*_{n,s} > 0, \\ \le 0, \text{if } x^*_{n,s} = 0, \end{cases} \end{aligned} \tag{12}$$

From (12), if subcarrier s is assigned to node n (i.e., $x^*_{n,s} > 0$), node n has the largest subcarrier sum-rate $R_{n,s} = \lambda_s$, which implies (11). ■

Although Proposition 1 gives a necessary condition for optimality, we cannot directly obtain an optimal solution because the conditions for X^* and (P^*, Q^*) are interdependent. Instead, we can have an intuition from (11) that each subcarrier s should be allocated to node n with the largest subcarrier sum-rate. Motivated by this intuition, we propose a resource allocation algorithm next.

IV. Proposed Resource Allocation Algorithm

In this section, we develop a solution that blends greedy subcarrier assignment and water-falling power allocation under per-node power constraints. We start with describing power allocation, and then consider subcarrier assignment.

A. Power Allocation

Given an optimal subcarrier assignment X^*, problem P is reduced to a power allocation problem that can be easily solved by the well-known *water-filling* power allocation at the BS and each node. Specifically, each node n allocates its uplink power

$p_{n,s}$ to the assigned subcarrier s as

$$p_{n,s} = \begin{cases} [\alpha_n - 1/u_{n,s}]^+, & \text{if } x^*_{n,s} = 1, \\ 0, & \text{if } x^*_{n,s} = 0, \end{cases} \tag{13}$$

where $[\cdot]^+ := \max\{\cdot, 0\}$ and α_n is a constant (called water level) satisfying $\sum_{s=1}^{S} p_{n,s} = P_n$. Similarly, the BS can optimally allocate the downlink power $q_{n,s}$ to subcarrier s for node n with $x^*_{n,s} = 1$ as

$$q_{n,s} = \begin{cases} [\alpha - 1/d_{n,s}]^+, & \text{if } x^*_{n,s} = 1, \\ 0, & \text{if } x^*_{n,s} = 0, \end{cases} \tag{14}$$

where α is a constant satisfying $\sum_{n=1}^{N}\sum_{s=1}^{S} q_{n,s} = P_{BS}$.

B. Subcarrier Assignment

Next, we design the subcarrier assignment algorithm that assigns subcarriers in an iterative manner, one-by-one, to a node with the largest subcarrier sum-rate. We take into account the dependency on the transmission power by re-allocating the power in each iteration.

Let $\mathcal{S}_n^{(k)}$ denote the set of subcarriers assigned to node n up to iteration k. If subcarrier s is assigned to a node, we denote the node by $n(s)$. Let $\mathcal{A}^{(k)}$ denote the set of assigned subcarriers up to iteration k, i.e., $\mathcal{A}^{(k)} = \cup_n \mathcal{S}_n^{(k)}$, and let $\mathcal{U}^{(k)} = \mathcal{S} \backslash \mathcal{A}^{(k)}$. We also use the superscript (k) to denote new power allocations, sum-rates, and downlink channel gains, calculated in iteration k. We use the superscript (0) to denote the initial value, e.g., $\mathcal{A}^{(0)} = \emptyset$.

In iteration k, we compute the potential subcarrier sum-rate given the subcarrier assignment of up to iteration $(k-1)$, and select a pair of (node, subcarrier) with the largest subcarrier sum-rate. To further elaborate, we first compute the subcarrier sum-rates $R_{n,s}^{(k)}$ for each node n as follows:

1) For each node n, re-allocate its uplink power $p_{n,s}^{(k)}$ using the water-filling algorithm (13) to the subcarriers that are assigned to node n or unassigned, i.e., $\mathcal{S}_n^{(k-1)} \cup \mathcal{A}^{(k-1)}$.
2) For each node n, reset its downlink channel gain $d_{n,s}^{(k)} = d_{n(s),s}$ for assigned subcarrier $s \in \mathcal{A}^{(k-1)}$, and $d_{n,s}^{(k)} = d_{n,s}$ for unassigned subcarrier $s \in \mathcal{U}^{(k-1)}$. Then, allocate all the BS power $q_{n,s}^{(k)}$ to node n with channel gain $d_{n,s}^{(k)}$, using the water-filling algorithm (14).
3) For each node n, compute the (potential) subcarrier sum-rate as

$$R_{n,s}^{(k)} = \log\left(1 + p_{n,s}^{(k)} u_{n,s}\right) + \log\left(1 + q_{n,s}^{(k)} d_{n,s}^{(k)}\right). \tag{15}$$

Given the per-node per-subcarrier sum-rate (15), among the unassigned subcarriers, we find a pair of (node, subcarrier) for subcarrier assignment according to the following procedures:

1) $(n^*, s^*) = \underset{n\in\mathcal{N}, s\in\mathcal{U}^{(k-1)}}{\arg\max} \; R_{n,s}^{(k)}$.
2) $\hat{x}_{n^*,s^*} \leftarrow 1$.
3) Update $\mathcal{U}^{(k)} \leftarrow \mathcal{U}^{(k-1)} \backslash \{s^*\}$, $\mathcal{A}^{(k)} \leftarrow \mathcal{A}^{(k-1)} \cup \{s^*\}$, and $\mathcal{S}_{n^*}^{(k)} \leftarrow \mathcal{S}_{n^*}^{(k-1)} \cup \{s^*\}$.

We repeat the above procedures S times and obtain the subcarrier assignment vector $\hat{X}$. Given $\hat{X}$, we finally allocate some powers for each node and the BS by (13) and (14), and obtain the power allocation vector $(\hat{P}, \hat{Q})$.

V. Numerical Results

In this section, we evaluate our resource allocation solution through numerical simulations. We use typical parameter values of LTE systems [10]: Each subcarrier has 15 kHz bandwidth and the noise power in each subcarrier is set to -130 dBm. The total transmission powers for the BS and each node n are set to 48 dBm and 24 dBm, respectively. Also, we assume zero (transmission and reception) antenna gain. For the path loss model, we use the Hata propagation model for urban environments.

We consider a time-slotted system. For channel model, we assume *i.i.d* Rayleigh block fading channel, where the channel gain for each subcarrier follows *i.i.d* Rayleigh distribution across users and time slots. We consider both symmetric and asymmetric channel models. In symmetric channel model, the uplink and downlink channel gains for each subcarrier are the same, and in asymmetric channel model, they are independently chosen.

For performance evaluation, we compare the following schemes:

- Upper Bound (UB): Performance upper bound obtained by separating uplink and downlink transmissions and ignoring the inter-node interference.
- Full-Duplex Optimal solution (FD-O): The optimal solution obtained by exhaustive search.
- Full-Duplex Proposed solution (FD-P): Proposed subcarrier assignment and power allocation algorithm.
- Full-Duplex Downlink optimal solution (FD-D): Assign each subcarrier to a node with the largest downlink channel gain and allocate (uplink and downlink) power according to the water-filling.
- Full-Duplex Uplink heuristic solution (FD-U): Assign subcarriers according to [7] and allocate (uplink and downlink) power according to the water-filling.
- Half-Duplex (HD): Downlink transmission and uplink transmission switch over time slots.

We compare the empirical performance of FD-P with the upper bound (UB) in symmetric channel environments. We set the number of nodes N to 50 and $D = 500$ m. Fig. 1 shows the performances of UB, FD-P, and HD as the number of subcarriers S increases from 10 to 100. For each value of S, the performance gap between UB and FD-P is negligible. This means that FD-P achieves empirically near-optimal performance. Also, FD-P achieves almost twice as large sum-rate as HD, implying that it can fully exploit the full-duplex capability.

We also evaluate FD-P in comparison with UB and FD-O in asymmetric channel environments. Due to the exponential complexity of FD-O, we assume a small-size network where N is set to 5. As shown in Fig. 2, FD-P achieves almost the same performance as FD-O, and thus FD-P is empirically near-optimal even in asymmetric channel environments. The performance gap between UB and FD-P ranges from 8% to

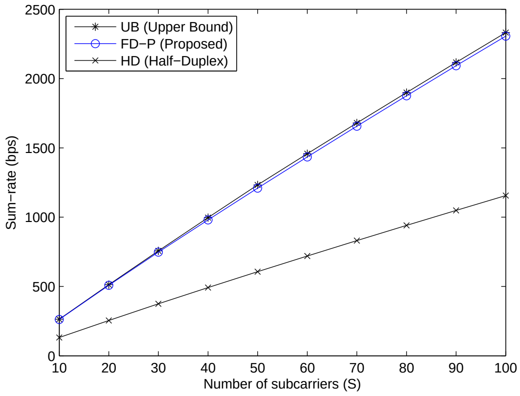

Fig. 1. Performance of UB, PD-P, and HD in symmetric channel when $N = 50$ and $D = 500$ m. The sum-rate of each scheme linearly increases with the number of subcarriers S due to the growing bandwidth. The performance gap between UB and FD-P is negligible, showing that FD-P achieves near-optimal performance.

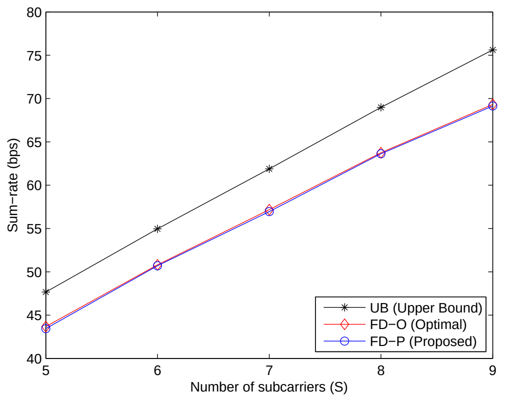

Fig. 2. Performance comparison in asymmetric channel environments when $N = 5$ and $D = 500$ m. FD-P achieves almost the same performance with FD-O, and thus it is empirically near-optimal.

9%, which is a huge increase compared to the symmetric channel case. In asymmetric channel environments, the full-duplex optimal solution is close neither to the uplink optimal solution nor to the downlink optimal solution. As a result, the separation of uplink and downlink transmissions results in a loose upper bound.

We now compare the performance of each scheme in large-size networks. We omit FD-O due to its high computational complexity and include UB and HD as an upper bound and a lower bound, respectively. Fig. 3 shows that FD-P outperforms both FD-D and FD-U substantially. The performance gain of FD-P over FD-D and FD-U increases with the number of subcarriers. As S increases from 10 to 100, FD-P shows the gain of 9.7% to 11.1% over FD-D and the gain of 13.6% to 17.4% over FD-U.

VI. Conclusion

Full-duplex transmission is a promising technology to boost the network capacity. In this paper, we have developed a new

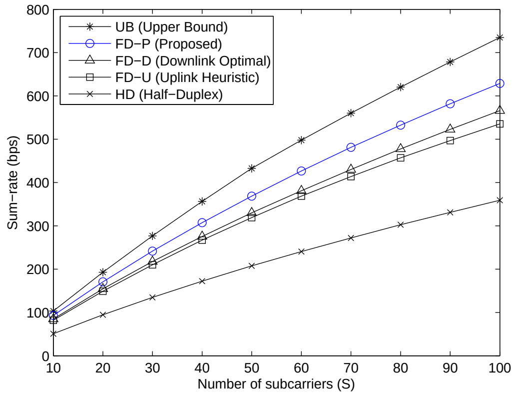

Fig. 3. Performance comparison in asymmetric channel environments when $N = 50$ and $D = 500$ m. FD-P outperforms FD-D and FD-U with a gain of 9.7% to 11.1% and 13.6% to 17.4%, respectively.

radio resource allocation algorithm for full-duplex OFDMA networks using a necessary condition for the optimal solution. The proposed algorithm assigns subcarriers to nodes in an iterative manner with low complexity. Through extensive numerical simulations, we demonstrate that our algorithm achieves near-optimal performance and outperforms other resource allocation schemes designed for half-duplex networks.

Acknowledgment

This research was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education, Science and Technology (No. 2012R1A1A2006171).

References

[1] Jung Il Choi, Mayank Jain, Kannan Srinivasan, Philip Levis, and Sachin Katti, "Achieving Single Channel Full Duplex Wireless Communication," in Proc. of ACM MobiCom, 2010.

[2] Mayank Jain, Jung Il Choi, Tae Min Kim, Dinesh Bharadia, Siddharth Seth, Kannan Srinivasan, Philip Levis, Sachin Katti, and Prasun Sinha, "Practical, real-time, full duplex wireless," in Proc. of ACM MobiCom, 2012.

[3] Dinesh Bharadia, Emily McMilin, and Sachin Katti, "Full-duplex Radio," in Proc. of ACM SIGCOMM, 2013.

[4] "3GPP Long Term Evolution," Available: http://www.3gpp.org/technologies/keywords-acronyms/98-lte.

[5] Young-June Choi, Suho Park, and Saewoong Bahk, "Multichannel Random Access in OFDMA Wireless Networks," IEEE Journal on Selected Areas in Communications, vol. 24, no. 3, 2006.

[6] Jiho Jang and Kwang Bok Lee, "Transmit Power Adaptation for Multiuser OFDM Sysmtes," IEEE Journal of Selected Areas in Communications, vol. 21, no. 2, 2003.

[7] Cho Yiu Ng and Chi Wan Sung, "Low Complexity Subcarrier and Power Allocation for Utility Maximization in Uplink OFDMA Systems," IEEE Transactions on Wireless Communications, vol. 7, no. 5, 2008.

[8] Jianwei Huang, Vijay G. Subramanian, Rejeev Agrawal, and Randall Berry, "Joint Scheduling and Resource Allocation in Uplink OFDM Systems for Broadband Wireless Access Networks," IEEE Journal on Selected Areas in Communications, vol. 27, no. 2, 2009.

[9] Stephen Boyd and Lieven Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.

[10] LTE Encyclopedia. https://sites.google.com/site/lteencyclopedia/lte-radio-link-budgeting-and-rf-planning.